

LEXICAL SCOPE

1. Motivation
2. Syntax
3. Environments & API
4. AST Evaluation
5. Operational Semantics.
6. Interpreter implementation

Lexical (Local) Identifiers.

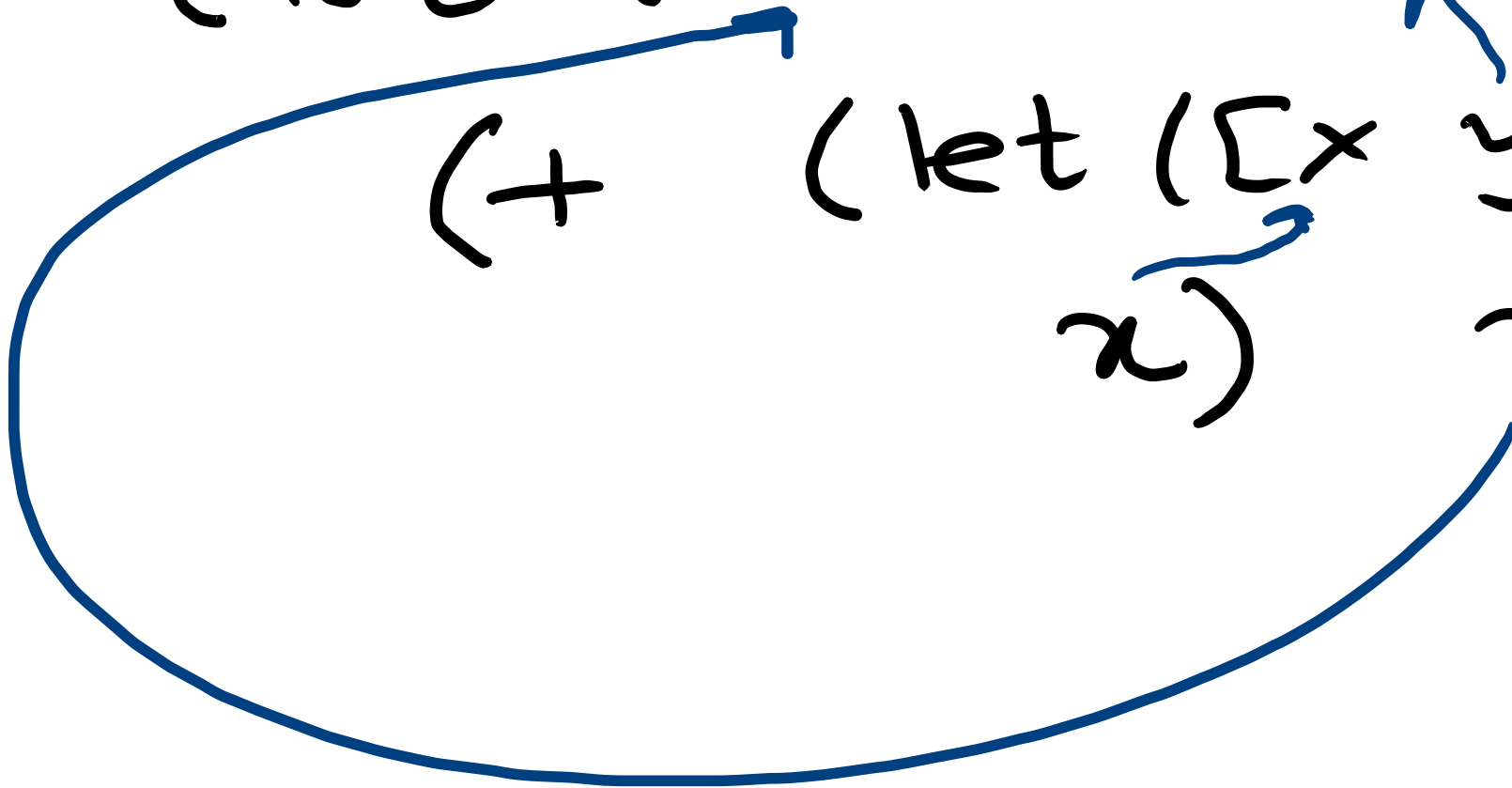
motivation: reuse of identifiers.

Scope: The area of the program
text where the identifier
is visible.

(let ([x 5] [y 3])

(+ (let ([x y])

x) x))



$(\lambda (x\ y))$

$(\text{if } (> x\ 0))$

$(\text{let } ([x\ 3])$

$(+ x\ y))$

$(\mathbb{Z}))$

free identifier

LEXICAL language

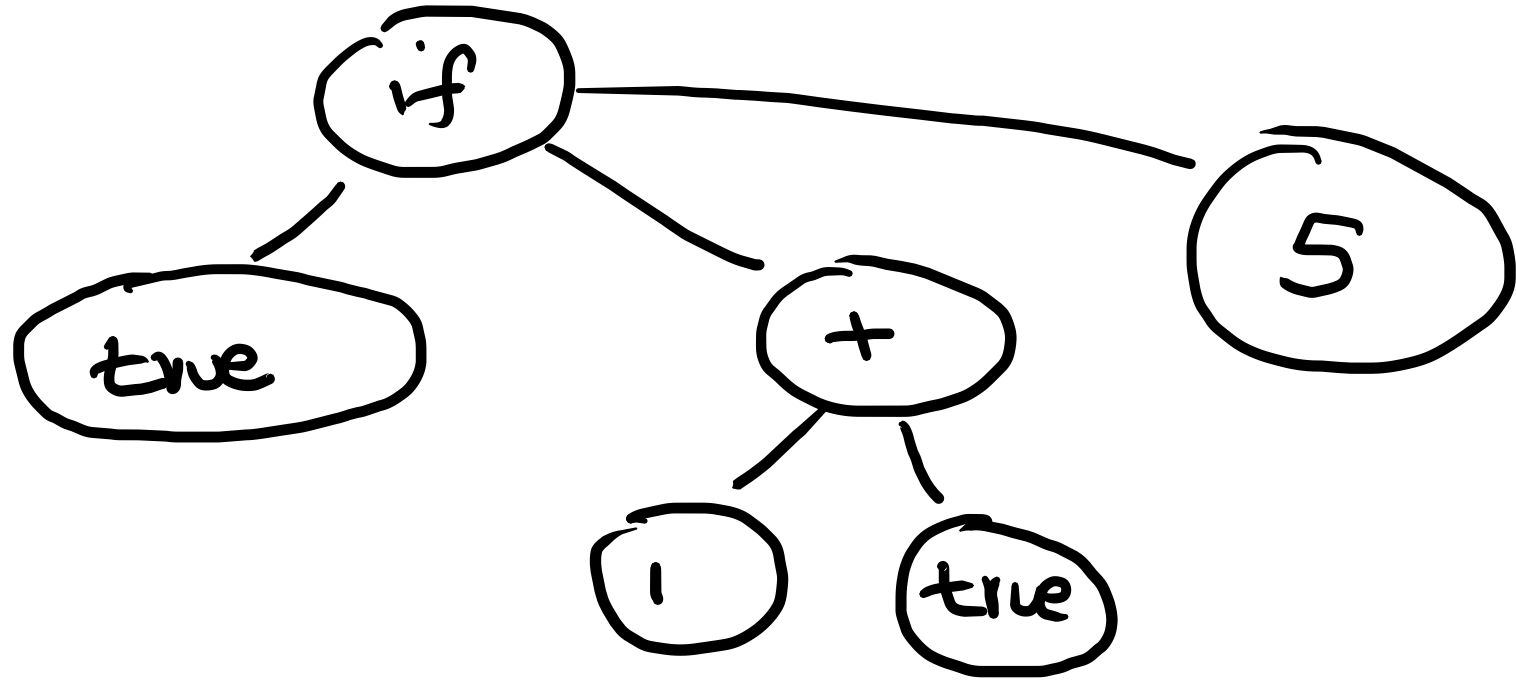
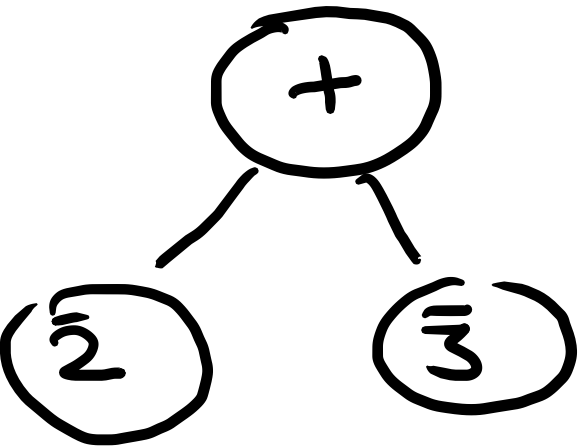
$x \in \text{Id}$: Identifiers

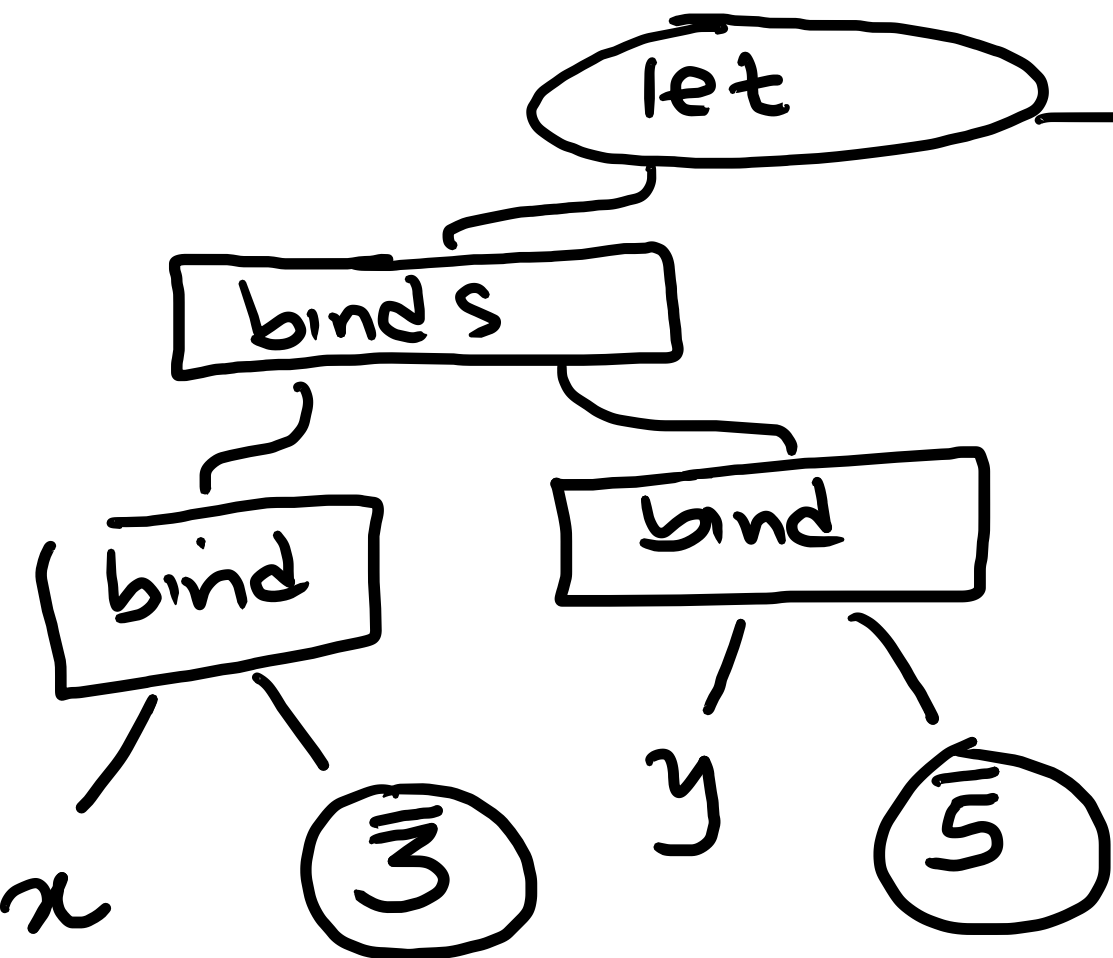
$v ::= b \mid n$ VALUES
 $b ::= \text{true} \mid \text{false}$ _____ BOOLEANS

$n ::= \mathbb{N}$

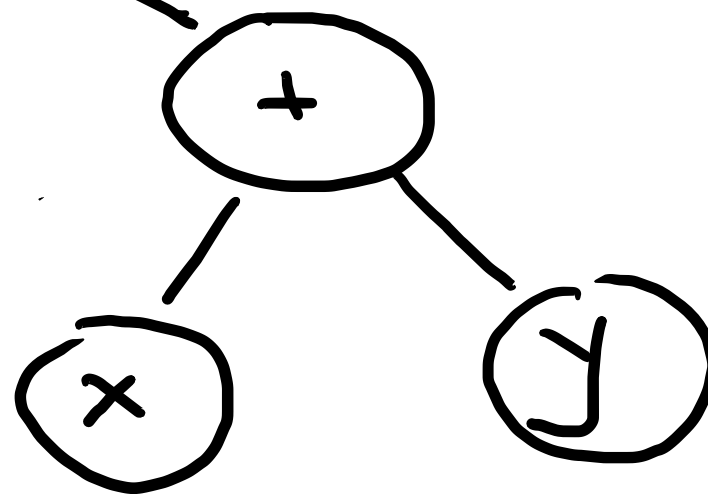
$e ::= v \mid + e e \mid \text{if } e e e$
 $\quad x \mid \text{let } (x e) \dots e \mid > e e$

Abstract Syntax Trees:



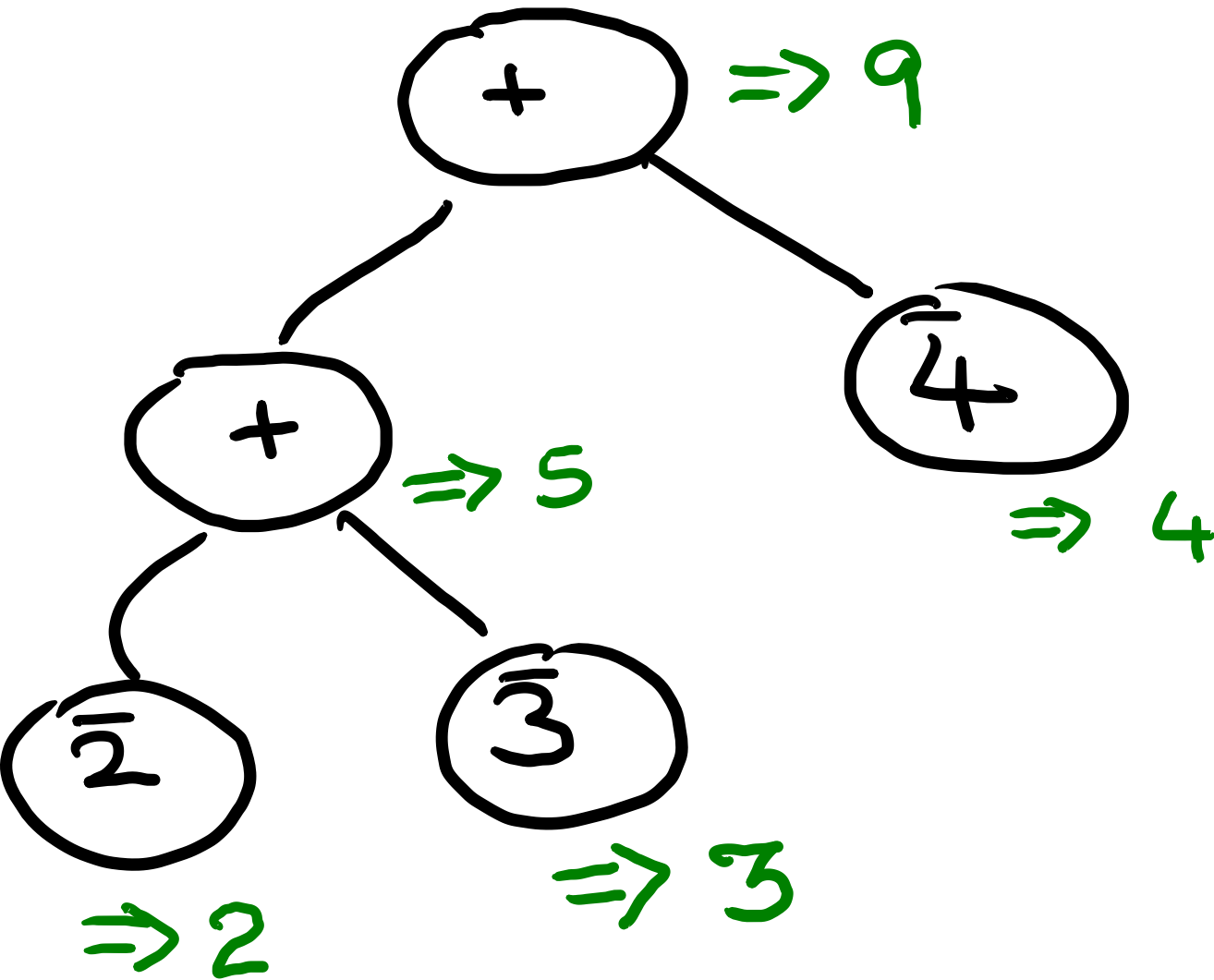


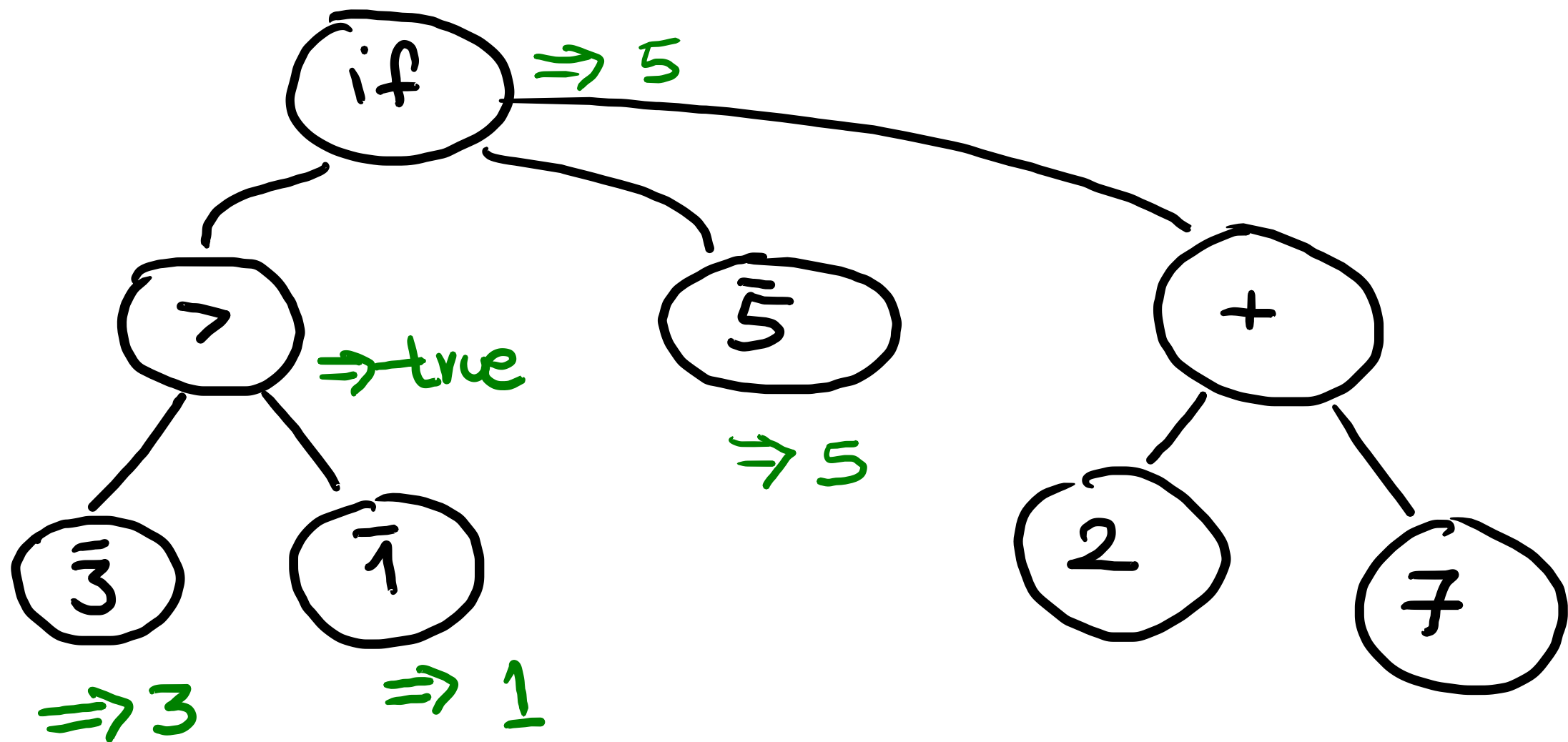
`let (x=3, y=5) in`
`x+y`



`(let ([x 3] [y 5])`
`(+ x y))`

Meaning of Programs





For ADDITION to IF/+, the
evaluation judgement was of the
form

$$e \Rightarrow v$$

What about identifiers

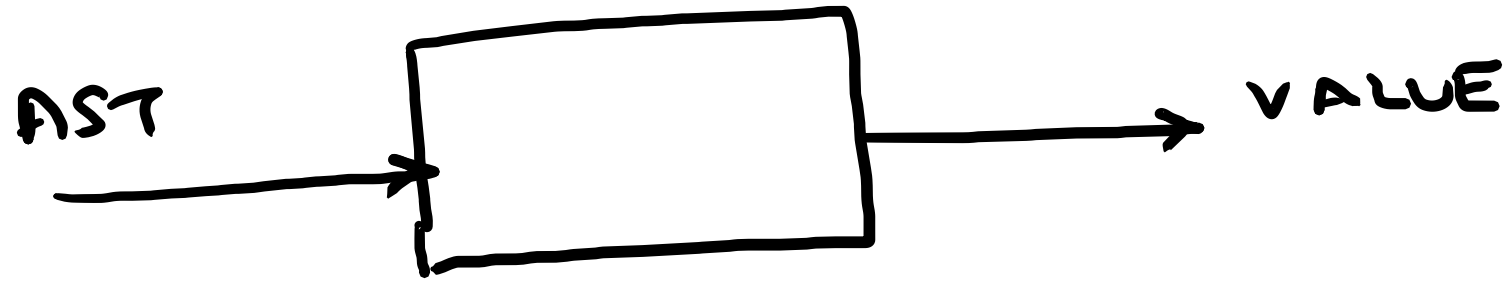
for lexical, we need to take into
account identifier bindings when
we evaluate.

$(+ x 3) \Rightarrow ?$

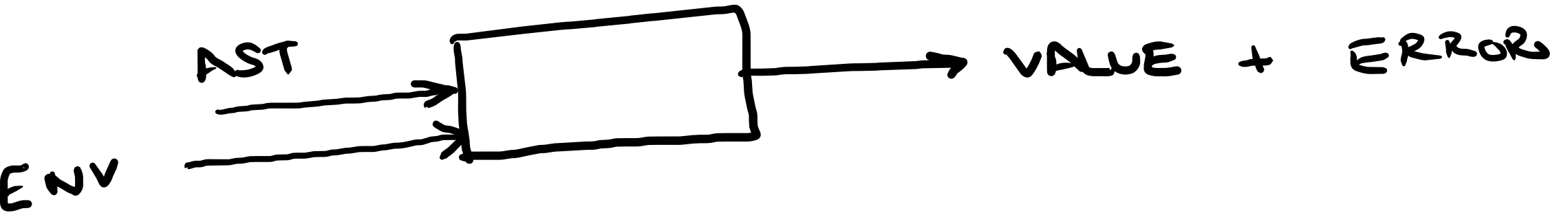
$\{x \mapsto 5\}$

$\vdash (+ x 3) \Rightarrow 8$

ADDITION



LEXICAL



Environments:

Finite Functions from identifiers to values.

$$\text{Env} = \text{Id} \xrightarrow{\text{fin}} \text{Val}$$

Examples: $\Gamma_1 = \{x \mapsto 3, y \mapsto \text{true}\}$

$$\Gamma_2 = \{y \mapsto 5, z \mapsto 7\}$$

$$\text{dom}(\Gamma_1) = \{x, y\}$$

$$\text{dom}(\Gamma_2) = \{y, z\}$$

Composition of Environments

$$\Gamma_1 = \{x \mapsto 3, y \mapsto \text{true}\}$$

$$\Gamma_2 = \{y \mapsto 5, z \mapsto 7\}$$

$$\text{lookup}(\Gamma, w) = \begin{cases} \Gamma(w) & \text{if } w \in \text{dom}(\Gamma) \\ \text{undef} & \text{o.w.} \end{cases}$$

$$\Gamma_1 \Gamma_2 = \{y \mapsto \text{true}, x \mapsto 3, z \mapsto 7\}$$

$$\text{dom}(\Gamma_1 \Gamma_2) = \text{dom}(\Gamma_1) \cup \text{dom}(\Gamma_2)$$

$$\text{lookup}(\Gamma_1 \Gamma_2, w) = \begin{cases} \Gamma_1(w) & \text{if } w \in \text{dom}(\Gamma_1) \\ \text{lookup}(\Gamma_2, w) & \text{o.w.} \end{cases}$$