Programming Languages and Compiler Design

Programming Language Semantics
Compiler Design Techniques

Yassine Lakhnech & Laurent Mounier

{lakhnech,mounier}@imag.fr

http://www-verimag.imag.fr/~lakhnech

http://www-verimag.imag.fr/~mounier.

Master of Sciences in Informatics at Grenoble (MoSIG)

Grenoble Universités

(Université Joseph Fourier, Grenoble INP)

Code Optimization

Objective (of this chapter)

- give some indications on general optimization techniques:
  - data-flow analysis
  - register allocation
  - software pipelining
  - etc.
- describe the main data structures used:
  - control flow graph
  - intermediate code (e.g., 3-address code)
  - Static Single Assignment form (SSA)
  - etc.
- see some concrete examples

But not a complete panorama of the whole optimization process

(e.g.: a real compiler, for a modern processor)

Objective of the optimization phase

Improve the ***efficiency*** of the target code, while preserving the source semantics.
efficiency $\rightarrow$ several (antagonist) criteria

- execution time
- size
- memory used
- energy consumption
- etc.

$\Rightarrow$ no optimal solution, no general algorithm
$\Rightarrow$ a bunch of optimization techniques:

- inter-dependant each others
- sometimes heuristic based

Two kinds of optimizations

Independant from the target machine

“source level” or “assembly level” pgm transformations:

- dead code elimination
- constant propagation, constant folding
- code motion
- common subexpressions elimination
- etc.

Dependant from the target machine

optimize the use of the hardware resources:

- machine instruction
- memory hierarchy (registers, cache, pipeline, etc.)
- etc.

Overview

1. Introduction
2. Some optimizations independant from the target machine
3. Some optimizations dependant from the target machine

Some optimizations independant from the target machine

Main principle

Input: initial intermediate code

Output: optimized intermediate code

Several steps:

1. generation of a control flow graph (CFG)
2. analysis of the CFG
3. transformation of the CFG
4. generation of the output code

Intraprocedural 3-address code (TAC)

“high-level” assembly code:

- binary logic and arithmetic operators
- use of temporary memory location `ti`
- assignments to variables, temporary locations
- a label is assigned to each instruction
- conditional jumps `goto`

Examples:

- `l:  x := y op x`
- `l:  x := op y`
- `l:  x := y`
- `l:  goto l’`
- `l:  if x oprel y goto l’`

Basic block (BB)

A maximal instruction sequence $S = i_1 . \cdots i_n$ such that:

- S execution is never "broken" by a jump
  $\Rightarrow$ no `goto` instruction in $i_1 . \cdots i_{n-1}$
- S execution cannot start somewhere in the middle
  $\Rightarrow$ no `label` in $i_2 . \cdots i_n$

$\Rightarrow$ execution of a basic bloc is atomic

Partition of a 3-address code BBs:

1. computation of Basic Block heads:
   1st inst., inst. target of a jump, inst. following a jump
2. computation of Basic Block tails:
   last inst, inst. before a Basic Block head

$\Rightarrow$ a single traversal of the TAC

Control Flow Graph (CFG)

A representation of how the execution may progress inside the TAC

$\rightarrow$ a graph (V, E) such that:

$$
\begin{aligned}
V &= \{B_i \mid B_i \text{ is a basic block}\} \\
E &= \{(B_i, B_j) \mid \\
&\quad \text{“last inst. of } B_i \text{ is a jump to 1st inst of } B_j\text{”} \vee \\
&\quad \text{“1st inst of } B_j \text{ follows last inst of } B_i \text{ in the TAC”}\}
\end{aligned}
$$

Example

Give the Basic Blocks and CFG associated to the following TAC sequence:

```
0. x := 1
1. y := 2
2. if c goto 6
3. x := x+1
4. z := 4
5. goto 8
6. z := 5
7. if d goto 0
8. z := z+2
9. r := 1
10 y := y-1
```

Optimizations performed on the CFG

Two levels:

Local optimizations:

- computed inside each BB
- BBs are transformed independent each others

Global optimizations:

- computed on the CFG
- transformation of the CFG:
  - code motion between BBs
  - transformation of BBs
  - modification of the CFG edges

Local optimizations

- algebraic simplification, strength reduction
  $\rightarrow$ replace costly computations by less expensive ones
- copy propagation
  $\rightarrow$ suppress useless variables
  (i.e., equal to another one, or equal to a constant)
- constant folding
  $\rightarrow$ perform operations between constants
- common subexpressions
  $\rightarrow$ suppress duplicate computations
  (already computed before)
- dead code elimination $\rightarrow$ suppress useless instructions
  (which do not influence pgm execution)

Example of local optimizations

Initial code:

```
a := x ** 2
b := 3
c := x
d := c * c
e := b * 2
f := a + d
g := e * f
```

Example of local optimizations

Algebraic simplification:

`a := x ** 2`	`a := x * x`
`b := 3`	`b := 3`
`c := x`	`c := x`
`d := c * c`	`d := c * c`
`e := b * 2`	`e := b << 1`
`f := a + d`	`f := a + d`
`g := e * f`	`g := e * f`

Example of local optimizations

Copies propagation:

```
a := x * x          a := x * x
b := 3              b := 3
c := x              c := x
d := c * c          d := x * x
e := b << 1         e := 3 << 1
f := a + d          f := a + d
g := e * f          g := e * f
```

Example of local optimizations

Constant folding:

`a := x * x`	`a := x * x`
`b := 3`	`b := 3`
`c := x`	`c := x`
`d := x * x`	`d := x * x`
`e := 3 << 1`	`e := 6`
`f := a + d`	`f := a + d`
`g := e * f`	`g := e * f`

Example of local optimizations

Elimination of common subexpressions:

`a := x * x`	`a := x * x`
`b := 3`	`b := 3`
`c := x`	`c := x`
`d := x * x`	`d := a`
`e := 6`	`e := 6`
`f := a + d`	`f := a + d`
`g := e * f`	`g := e * f`

Example of local optimizations

Copies propagation:

`a := x * x`	`a := x * x`
`b := 3`	`b := 3`
`c := x`	`c := x`
`d := a`	`d := a`
`e := 6`	`e := 6`
`f := a + d`	`f := a + a`
`g := e * f`	`g := 6 * f`

Example of local optimizations

Dead code elimination (+ strength reduction):

```
a := x * x        a := x * x         a:= x * x
b := 3
c := x
d := a
e := 6
f := a + a        f := a + a         f := a << 1
g := 6 * f        g := 6 * f         g := 6 * f
```

Local optimization: a more concrete example

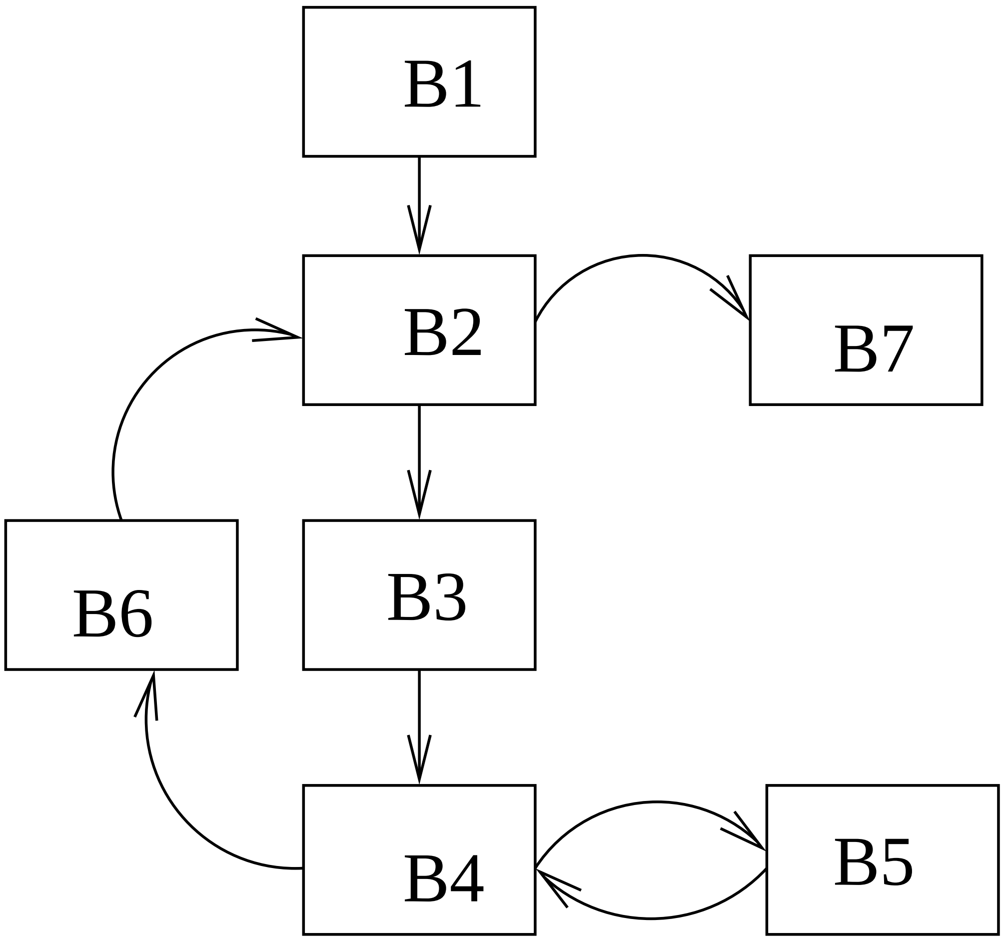
Inital source program: addition of matrices

```
for (i=0 ; i < 10 ; i ++)
     for (j=0 ; j < 10 ; j++)
        S[i,j] := A[i,j] + B[i,j]
```

Basic blocks:

B1:	i := 0
B2:	if i $>$ 10 goto B7
B3:	j := 0
B4:	if j $>$ 10 goto B6
$\boxed{\text{B5}}$	
B6:	i := i + 1
	goto B2
B7: end	

Control Flow Graph



Inital Block B5

```
B5:   t1 := 4 * i
      t2 := 40 * j
      t3 := t1 + t2
      t4 := A[t3]
      t5 := 4 * i
      t6 := 40 * j
      t7 := t5 + t6
      t8 := B[t7]
      t9 := t4 + t8
      t10:= 4 * i
      t11:= 40 * j
      t12:= t10 + t11
      S[t12] := t9
      j := j + 1
      goto B4
```

Optimization of B5 (1/4)

```
B5:  t1 := 4 * i        t8 := B[t7]
     t2 := 40 * j       t9 := t4 + t8
     t3 := t1 + t2      t10:= 4 * i
     t4 := A[t3]        t11:= 40 * j
     t5 := 4 * i        t12:= t10 + t11
     t6 := 40 * j       S[t12] := t9
     t7 := t5 + t6      j := j + 1
                        goto B4
```

A same value is assigned to temporary locations t1, t5, t10

Optimization of B5 (2/4)

```
B5:   t1 := 4 * i          t8 := B[t7]
      t2 := 40 * j         t9 := t4 + t8
      t3 := t1 + t2        t11:= 40 * j
      t4 := A[t3]          t12:= t1 + t11
      t6 := 40 * j         S[t12] := t9
      t7 := t1 + t6        j := j + 1
                           goto B4
```

A same value is assigned to temporary locations t2, t6, t11

Optimization of B5 (3/4)

```
B5:   t1 := 4 * i
      t2 := 40 * j
      t3 := t1 + t2
      t4 := A[t3]
      t7 := t1 + t2
      t8 := B[t7]
      t9 := t4 + t8
      t12:= t1 + t2
      S[t12] := t9
      j := j + 1
      goto B4
```

A same value is assigned to temporary locations t3, t7, t12

Optimization of B5 (4/4): the final code obtained

```
B5:   t1 := 4 * i
      t2 := 40 * j
      t3 := t1 + t2
      t4 := A[t3]
      t8 := B[t3]
      t9 := t4 + t8
      S[t3] := t9
      j := j + 1
      goto B4
```

Global optimizations

Global optimization: the principle

Typical examples of global optimizations:

- constant propagation trough several basic blocks
- elimination of global redundancies
- code motion: move invariant computations outside loops
- dead code elimination

How to “extrapolate” local optimizations to the whole CFG ?

1. associate (local) properties to entry/exit points of BBs
   (set of active variables, set of available expressions, etc.)
2. propagate them along CFG paths
   $\rightarrow$ enforce consistency w.r.t. the CFG structure
3. update each BB (and CFG edges) according to these global properties

$\Rightarrow$ a possible technique: data-flow analysis

Data-flow analysis

Static computation of data related properties of programs

- (local) properties φ_i associated to some pgm locations i

- set of data-flow equations:
  $\rightarrow$ how φ_i are transformed along pgm execution
  Rks:
  - forward vs backward propagation (depending on φ_i)
  - cycles inside the control flow $\Rightarrow$ fix-point equations !

- a solution of this equation system:
  $\rightarrow$ assigns "globaly consistent" values to each φ_i
  Rk: such a solution may not exist ...

- decidability may require abstractions and/or approximations

Example: elimination of redundant computations

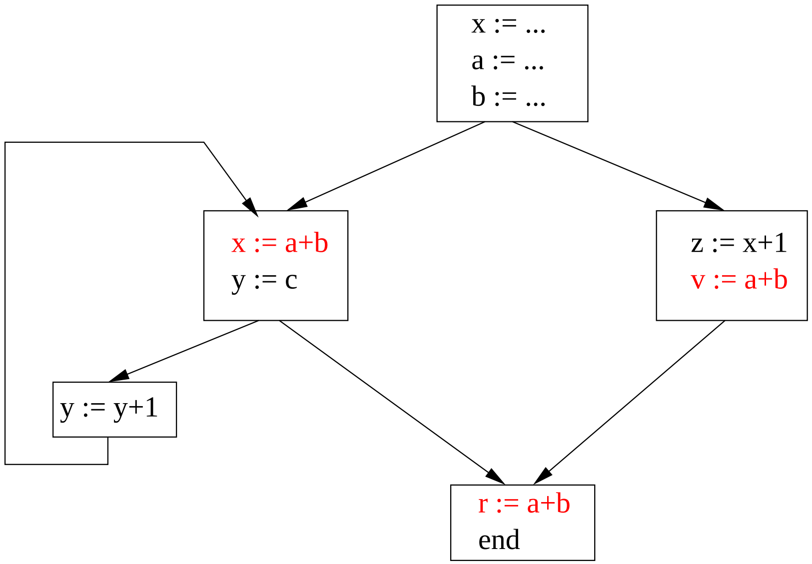
An expression e is redundant at location i iff

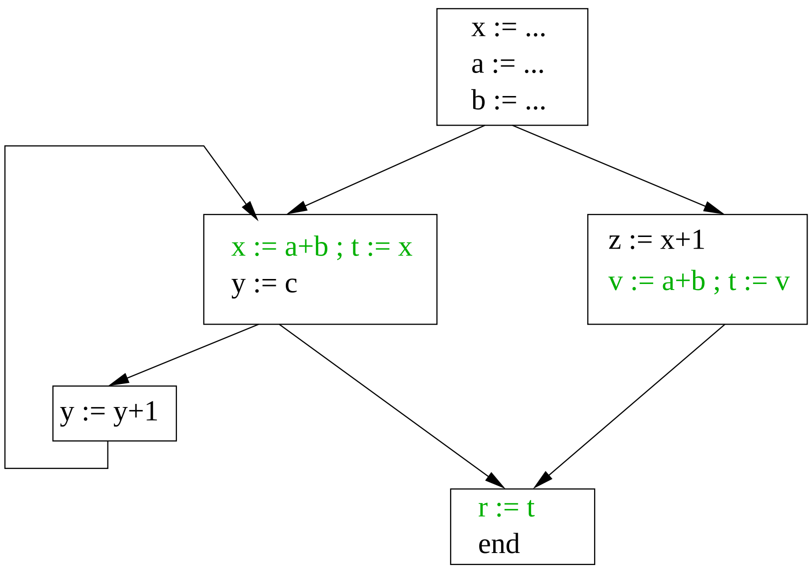
- it is computed at location i
- this expression is computed on every path going from the initial location to location i
  Rk: we consider here syntactic equality
- on each of these paths: operands of e are not modified between the last computation of e and location i

Optimization is performed as follows:

1. computation of available expressions (data-flow analysis)
2. $x := e$ is redundant at loc i if e is available at i
3. $x := e$ is replaced by $x := t$
   (where t is a temp. memory containing the value of e)

Elimination of redundant computation: an example





Data-flow equations for available expressions (1/2)

For a basic block b, we note:

- $In(b)$: available expressions when entering b
- $Kill(b)$: expressions made non available by b (because an operand of e is modified by b)
- $Gen(b)$: expressions made available by block b (computed in b, operands not modified afterwards)
- $Out(b)$: available expressions when exiting b

$$Out(b) = (In(b) \setminus Kill(b)) \cup Gen(b) = F_b(In(b))$$

F_b = transfer function of block b

Data-flow equations for available expressions (2/2)

How to compute $In(b)$?

- if b is the initial block:

$$In(b) = \emptyset$$

- if b is not the initial block:
  An expression e is available at its entry point iff it is available at the exit point of each predecessor of b in the CFG

$$In(b) = \bigcap_{b' \in Pre(b)} Out(b')$$

$\Rightarrow$ forward data-flow analysis along the CFG paths

Q: cycles inside the CFG $\Rightarrow$ fix-points computations
greatest vd least solutions ?

Solving the data-flow equations (1/2)

Let $(E, \leq)$ a partial order.

- For $X \subseteq E, a \in E$:
  - a is an upper bound of X if $\forall x \in X.\ x \leq a$
  - a is a lower bound of X if $\forall x \in X.\ a \leq x$
- The least upper bound (*lub*, $\sqcup$) is the smallest upper bound
- The great lower bound (*glb*, $\sqcap$) is the largest lower bound
- $(E, \leq)$ is a lattice if every subset of E admits a *lub* and a *glb*.
- A function $f : 2^E \to 2^E$ is monotonic if:

$$\forall X, Y \subseteq E \quad X \leq Y \ \Rightarrow \ f(X) \leq f(Y)$$

- $X = \{x_0, x_1, \ldots x_n, \ldots\} \subseteq E$ is an (increasing) chain if $x_0 \leq x_1 \leq \ldots x_n \leq \ldots$
- A function $f : 2^E \to 2^E$ is ($\sqcup$-)continuous if $\forall$ increasing chain X, $f(\sqcup X) = \sqcup f(X)$

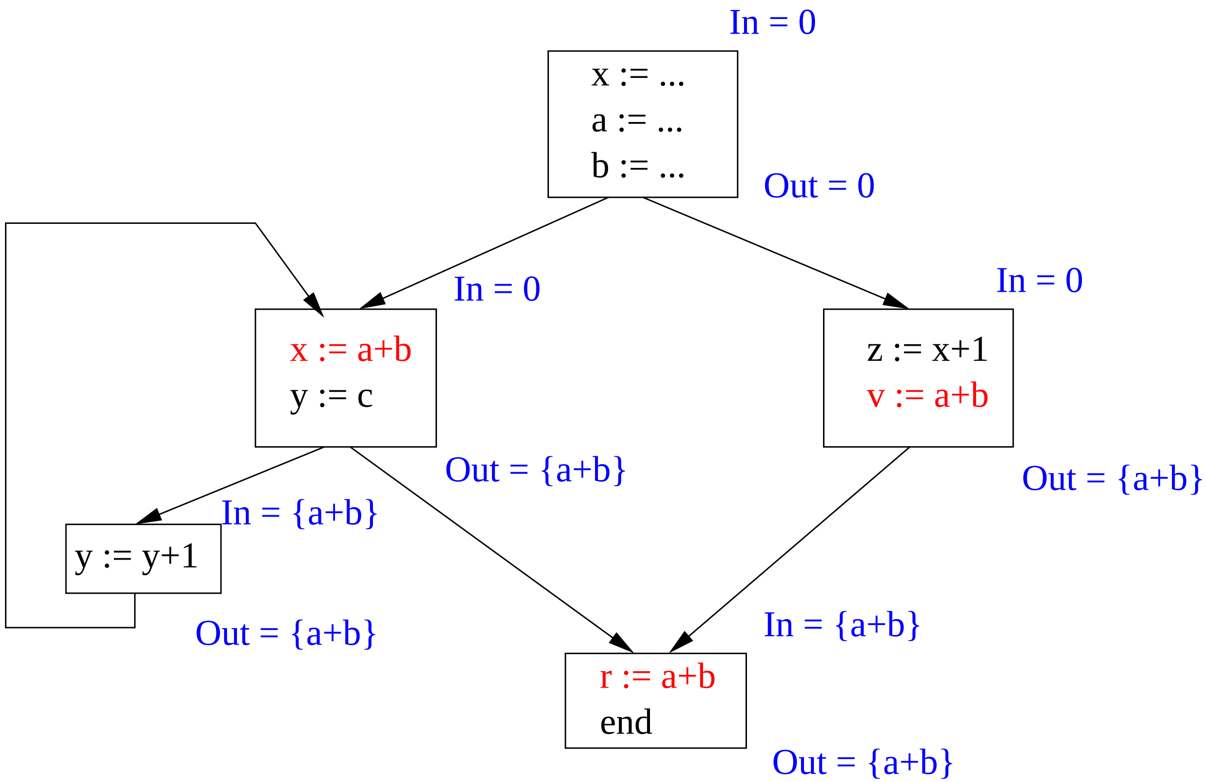
Solving the data-flow equations (2/2)

Fix-point equation: solution ?

- properties are finite sets of expressions $\mathcal{E}$
- $(2^{\mathcal{E}}, \subseteq)$ is a complete lattice
  $\perp$: least element, $\top$: greatest element
  $\sqcap$: greatest lower bound, $\sqcup$: least upper bound
- data-flow equations are defined on monotonic and continuous operators ($\cup$, $\cap$) on $(2^{\mathcal{E}}, \subseteq)$
- Kleene and Tarski theorems:
  - the set of solution is a complete lattice
  - the greatest (resp. least) solution can be obtained by successive iterations w.r.t. the greatest (resp. least) element of $2^{\mathcal{E}}$

$$\text{lfp}(f) = \sqcup\{f^i(\perp) | i \in \mathbb{N}\} \qquad \text{gfp}(f) = \sqcap\{f^i(\top) | i \in \mathbb{N}\}$$

Back to the example



Generalization

- Data-flow properties are expressed as finite sets associated to entry/exit points of basic blocs: `In`(b), `Out`(b)

- For a forward analysis:
  - property is "false" ($\perp$) at entry of initial block
  - $\texttt{Out}(b) = F_b(\texttt{In}(b))$
  - `In`(b) depends on `Out`(b'), where $b' \in Pred(b)$
    ($\sqcap$ for "$\forall$ paths", $\sqcup$ for "$\exists$ path")

- For a backward analysis:
  - property is "false" ($\perp$) at exit of final block
  - $\texttt{In}(b) = F_b(\texttt{Out}(b))$
  - `Out`(b) depends on `In`(b'), where $b' \in Succ(b)$

Data-flow equations: forward analysis

Forward analysis, least fix-point	$$\begin{array}{rcl} \mathtt{In}(b) & = & \begin{cases} \perp & \text{if b is initial} \\ \bigsqcup_{b' \in Pre(b)} \mathtt{Out}(b') & \text{otherwise.} \end{cases} \\ \mathtt{Out}(b) & = & F_b(\mathtt{In}(b)) \end{array}$$
Forward analysis, greatest fix-point	$$\begin{array}{rcl} \mathtt{In}(b) & = & \begin{cases} \perp & \text{if } b \text{ is initial} \\ \bigsqcap_{b' \in Pre(b)} \mathtt{Out}(b') otherwise. \end{cases} \\ \mathtt{Out}(b) & = & F_b(\mathtt{In}(b)) \end{array}$$

Data-flow equations: backward analysis

Backward analysis, least fix-point	$\texttt{Out}(b) = \begin{cases} \bot & \text{if } b \text{ is final} \\ \bigsqcup_{b' \in Succ(b')} \texttt{In}(b') otherwise. \end{cases}$ $\texttt{In}(b) = F_b(\texttt{Out}(b))$
Backward analysis, greatest fix-point	$\texttt{Out}(b) = \begin{cases} \bot & \text{if } b \text{ is final} \\ \bigsqcap_{b' \in Succ(b)} \texttt{In}(b') otherwise. \end{cases}$ $\texttt{In}(b) = F_b(\texttt{Out}(b))$

Active Variable

- A variable `x` is inactive at location `i` if it is not used in every CFG-path going from `i` to `j`, where `j` is:
  - either a final instruction
  - or an assignement to `x`.

- An instruction `x := e` at location `i` is useless if `x` is inactive at location `i`.

$\Rightarrow$ useless instuctions can be removed . . .

Rk: used means

“in a right-hand side assignment or in a branch condition”.

Data-flow analysis for inactive variables

We compute the set of active variables . . .

Local analysis

$\texttt{Gen}(b)$ is the set of variables x s.t. x is used in block b, and, in this block, any assignement to x happens after the (first) use of x.

$\texttt{Kill}(i)$ is the set of variables x assigned in block b.

Global analysis : backward analysis, $\exists$ a CFG-path (least solution)

- $$\begin{array}{rcl} \texttt{Out}(b) & = & \bigcup_{b' \in Succ(b)} \texttt{In}(b') \\ \texttt{In}(b) & = & (\texttt{Out}(b) \setminus \texttt{Kill}(b)) \cup \texttt{Gen}(b) \end{array}$$
- $\texttt{Out}(b) = \emptyset$ if b is final.

Computation of functions Gen and $Kill$

Recursively defined on the syntax of a basic bloc B:

$\texttt{B} ::= \varepsilon \mid \texttt{B ; x := a} \mid \texttt{B ; if b goto l} \mid \texttt{B ; goto l}$

$Gen(B)$	$=$	$Gen_l(\texttt{B}, \emptyset)$
$Kill(B)$	$=$	$Kill_l(\texttt{B}, \emptyset)$
$Gen_l(\texttt{B ; x := a}, X)$	$=$	$Gen_l(\texttt{B}, X \setminus \{\texttt{x}\} \cup \mathsf{Used}(\texttt{a}))$
$Gen_l(\texttt{B ; if b goto l}, X)$	$=$	$Gen_l(\texttt{B}, X \cup \mathsf{Used}(\texttt{b}))$
$Gen_l(\texttt{B ; goto l}, X)$	$=$	$Gen_l(\texttt{B}, X)$
$Gen_l(\varepsilon, X)$	$=$	X
$Kill_l(\texttt{B ; x := a}, X)$	$=$	$Kill_l(\texttt{B}, X \cup \{\texttt{x}\})$
$Kill_l(\texttt{B ; if b goto l}, X)$	$=$	$Kill_l(\texttt{B}, X)$
$Kill_l(\texttt{B ; goto l}, X)$	$=$	$Kill_l(\texttt{B}, X)$
$Kill_l(\varepsilon, X)$	$=$	X

Used(e): set of variables appearing in expression e

Removal of useless instructions

1. Compute the sets $In(B)$ and $Out(B)$ of active variables at entry and exit points of each blocks.
2. Let $F : Code \times 2^{Var} \to Code$
   $F(b, X)$ is the code obtained when removing useless assignments inside b, assuming that variables of X are active at the end of b execution.

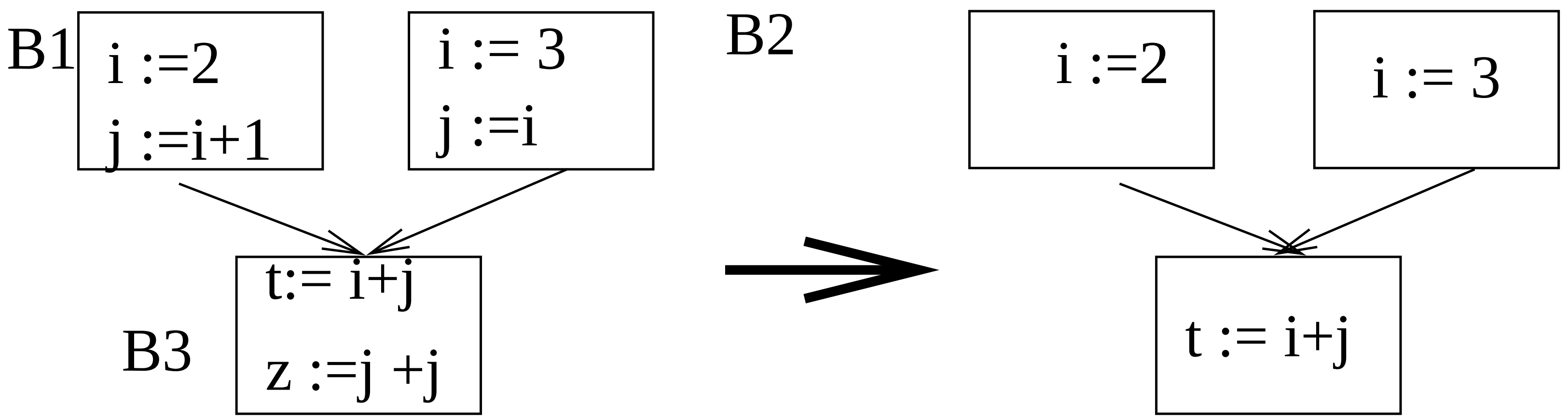
$$
\begin{array}{lcl}
F(\mathtt{B}\ ;\ \mathtt{x}\ := \mathtt{a}, X) & = & \begin{cases} F(B, X) & \text{if } x \notin X \\ F(B, (X \setminus \{x\}) \cup \mathtt{Used}(a)); x := a & \text{if } x \in X \end{cases} \\
F(\ \mathtt{B}\ ;\ \mathtt{if\ b\ goto\ l}, X) & = & F(B, X \cup \mathtt{Used}(b)); \mathtt{if\ b\ goto\ l} \\
F(\mathtt{B}\ ;\ \mathtt{goto\ l}, X) & = & F(B, X); \mathtt{goto\ l} \\
F(\epsilon, X) & = & \epsilon
\end{array}
$$

3. Replace each block B by $F(B, Out(B))$.

Rk: this transformation may produce new inactive variables . . .

Constant propagation

Example:



- A variable is constant at location `l` if its value at this location can be computed at compilation time.
- At exit point of B1 and B2, `i` and `j` are constants
- At entry point of B3, `i` is not constant, `j` is constant.

Constant propagation: the lattice

- Each variable takes its value in $D = \mathbb{N} \cup \{\top, \bot\}$, where:
  - $\top$ means “non constant value”
  - $\bot$ means “no information”

- Partial order relation $\leq$:
  if $v \in D$ then $\bot \leq v$ and $v \leq \top$.

- The least upper bound $\sqcup$:
  for $x \in D$ and $v_1, v_2 \in \mathbb{N}$

$x \sqcup \top = \top$	$x \sqcup \bot = x$	$v_1 \sqcup v_2 = \top$if $v_1 \neq v_2$	$v_1 \sqcup v_1 = v_1$

Rk: relations $\leq$ is extended to functions $Var \to D$

$$f1 \leq f2 \text{ iff } \forall x. f1(x) \leq f2(x)$$

Constant propagation: data-flow equations

- property at location `l` is a function $Var \to D$.
- Forward analysis:

$$
\begin{array}{rcl}
In(b) & = & \begin{cases} \lambda x.\bot & \text{if } b \text{ is initial,} \\ \displaystyle\bigsqcup_{b' \in Pred(b)} Out(b') & \text{otherwise} \end{cases} \\
Out(b) & = & F_b(In(b))
\end{array}
$$

Transfer function F_b ?

a basic block = sequence of assignements

$$\mathsf{b} \quad ::= \quad \epsilon \mid \mathtt{x{:}{=}e}\ ;\ \mathsf{b}$$

F_b defined by syntactic induction:

$$
\begin{array}{rcl}
F_{\mathtt{x:=e}\,;\,\mathsf{b}}(f) & = & F_{\mathsf{b}}(f[x \mapsto f(e)]) \quad \text{(assuming variable initialization)} \\
F_\epsilon(f) & = & f
\end{array}
$$

Pgm transformation:

$\forall$ block b, $f \in In(b)$, $f(\mathsf{e}) = v \Rightarrow$ `x:=e` replaced by `x:=v`

Exercise

Constant propagation can be viewed as abstraction of the standard semantics where expressions values are interpreted other domain D

1. Write this abstract semantics for the `while` language in an operational style (relation $\longrightarrow_{\#}$)
2. Define a program transformation which removes useless computations (i.e., computations between constant operands)
3. Give the equations which express the correctness of this transformation

Another example of data-flow analysis

A computation of an expression e can be anticipated at loc. p iff:

- all paths from p contains a location p_i s.t. e is computed at p_i
- e operands are not modified between p and p_i

Example:

```
if (x>0)
      x = i + j;
else
      repeat y = (i + j) * 2; x := x+1 ; until x>10
```

can be changed to

```
tmp = i + j;
if (x>0)
      x = tmp;
else
      repeat y = tmp * 2; x := x+ 1 ; until x>10
```

Application: moving invariants outside loops

Interprocedural analysis

```
main()
{
  int i,j ;
  void f(){
    int x,y ;
    y = i+j ; x = y ;
  }
  i = 0 ;
  f() ;
  j = 1;
}
```

- a dedicated basic block B_{call} for the `call` instruction
- $In(B_{call}) = In(B_{f_{in}}), Out(B_{call}) = Out(B_{f_{out}})$

Rks:

- static binding is be assumed
- parameters ?

Exercice: Computation of active variables

Control-flow analysis

$\rightarrow$ retrieve program control structures from the CFG ?
Application: loop identification

$\Rightarrow$ use of graph-theoretic notions:

- dominator, dominance relation
- strongly connected components

Rk1: most loops are easier to identify at syntactic level, but:

- use of `goto` instruction still allowed in high-level languages
- optimization performed on intermediate representations (e.g., CFG)

Rk2: other approaches can be used to identify loops . . .

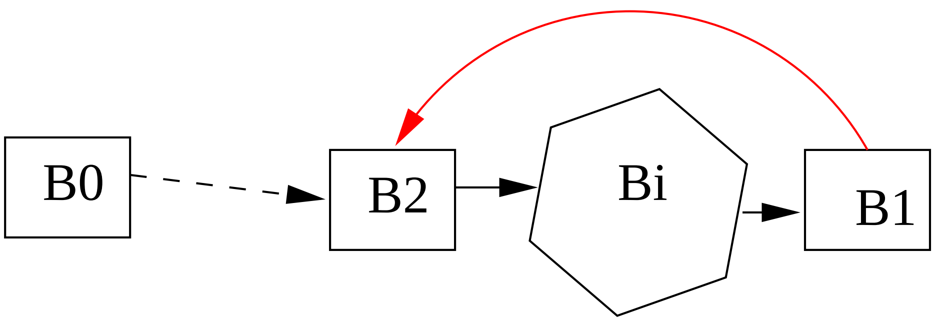
Loop identification

Node B_1 is a dominator of B_2 ($B_2 \leq B_1$) iff every path from the entry block to B_2 goes through B_1. $Dom(B) = \{B_i | B_i \leq B\}$.

An edge (B_1, B_2) is a loop back edge iff $B_2 \leq B_1$

To find "natural loops":

1. find a back edge (B_1, B_2)
2. find $Dom(B_2)$
3. find blocks $B_i \in Dom(B_2)$ s.t. there is a path from B_i to B_2 not containing B_1.



Some machine level optimization techniques

Register Allocation

Pb:

- expression operands are much efficiently accessed when liying in registers (instead of RAM)
- the “real” number of registers is finite (and usually small)

⇒ register allocation techniques:

- assigns a register to each operand (variable, temporary location)
- performs the memory exchange (`LD`, `ST`) when necessary
- optimality ?

Several existing techniques:

- optimal code generation for arithmetic expressions
- graph-coloring techniques (more general case)
- etc.

Code generation for arithmetic expressions: example

code generation for `(a+b) - (c - (d+e))`
with 2 registers, and instruction format = `OP Ri, Ri, X` (where `X=Ri` or `X=M[x]`)

Solution 1: one register needs to be saved

```
LD R0, M[a]
ADD R0, R0, M[b]
LD R1, M[d]
ADD R1, R1, M[e]
ST R1, M[t1]        ! register R1 needs to be saved ...
LD R1, M[c]
SUB R1, R1, M[t1]
SUB R0, R0, R1
```

Solution 2: no register to save

```
LD R0, M[c]
LD R1, M[d]
ADD R1, R1, M[e]
SUB R0, R0, R1
LD R1, M[a]
ADD, R1, R1, M[b]
SUB, R1, R1, R0
```

Code generation for arithmetic expressions: principle

Evaluation of `e1 op e2` , assuming:

- r registers are available, evaluation of `ei` requires r_i registers
- intsruction format is “op reg, reg, ad” where “ad” is a register or a memory location

Several cases:

- $\boxed{r_1 > r_2\text{:}}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 \geq r_2 \Rightarrow r_1 - 1$ registers are enough for `e2`
  - $\Rightarrow \boxed{r_1 - r}$ register allocations are required
- $\boxed{r_1 = r_2\text{:}}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 < r_2, \Rightarrow r_2$ (=r_1) registers required for `e2`
  - $\Rightarrow \boxed{r_1 + 1 - r}$ register allocations are required
- $\boxed{r_1 < r_2\text{:}}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 < r_2, \Rightarrow r_2$ ($> r_1$) registers required for `e2`
  - $\Rightarrow r_2 + 1 - r$ register allocations are required
  - $r_2 - r$ allocations are enough if e2 is evaluated first !

A two-phase algorithm

Step 1: each AST node is labeled with the number of registers required for its evaluation

rNb : $Aexp \to \mathbb{N}$ (rNb$(\texttt{e})$ is the number of registers required to evaluate e)

$$\text{rNb}(\texttt{e}) = \begin{cases} 1 & \text{if } e \quad \text{is a left leaf} \\ 0 & \text{if } e \quad \text{is a right leaf} \end{cases}$$

$$\text{rNb}(\texttt{e1 op e2}) = \begin{cases} max(\text{rNb}(\texttt{e}_1), \text{rNb}(\texttt{e}_2)) & \text{if rNb}(\texttt{e}_1) \neq \text{rNb}(\texttt{e}_2) \\ \text{rNb}(\texttt{e}_1) + 1 & \text{if rNb}(\texttt{e}_1) = \text{rNb}(\texttt{e}_2) \end{cases}$$

Step 2: “optimal” code generation using these labels (exercice)

$\to$ for a binary node `e1 op e2`:

- evaluate the more register demanding sub-expression first
- write the result in a register Ri (save one if necessary)
- evaluate the other sub-expression, write the result in a register Rj
- generate `OP, Ri, Ri, Rj`

A more general technique

1. Intermediate code is generated assuming ∞ numbers of "symbolic" registers S_i
2. Assign a real register R_i to each symbolic register s.t.
   - if R_i is assigned to S_i, R_j is assigned to S_j
   - then Lifetime$(S_i) \cap$ lifetime$(S_j) \neq \emptyset \Rightarrow R_i \neq R_j$

   where Lifetime(S_i): sequences of pgm location where S_i is active

How to ensure this condition ?

Collision graph G_C:

- Nodes denote lifetime symbolic registers: $N_i = (S_i,$ Lifetime$(S_i))$
- Edges are the set $\{((S_1, L_1), (S_2, L_2) \mid L_1$ and L_2 overlap$\}$

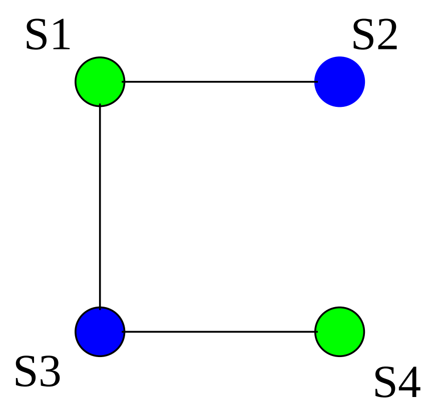
$\Rightarrow$ register allocation with k real register = k-coloring problem of G_C

(i.e., assign a distinct *colour* to each pair of adjacent nodes)

Example 1

```
S1 := e1
S2 := e2
...
... S2 ...                  S2 used
S3 := S1+S2                 S1 and S2 used
...
S4 := S1*5                  S1 used
... S4 ...                  S4 used
... S3 ...                  S3 used
```

Collision Graph:



Can be colored with 2 colors $\Rightarrow$ 2 real registers are enough . . .

k-coloring in practice ? (1)

When $k > 2$, this problem is NP-complete ...

An efficient heuristic:

Repeat:

if exists a node N of G_C such that degree$(N) < k$

(N can receive a distinct colour from all its neighbours)

remove N (and corresponding edges) from G_C and push it on a stack S

else (G_C is assumed to be non k-colourable)

choose a node N **(1)**

remove N from G_C **(2)**

until G_C is empty
While S is not empty

pop a node from S

add it to G, give it a colour not used by one of its neighbours

Rk: this algo may sometimes miss k-colorable graphs ...

k-coloring in practice ? (2)

What happens when there is no node of degree < k ?

(1) choose a node N to remove:
$\rightarrow$ high degree in G_C, not corresponding to an inner loop, etc.
(2) remove node N:
$\rightarrow$ save a register into memory before (register spilling)

Several attempts to improve this algorithm:

node coalescing:

`S1 := S2`, Lifetime$(S1) \cap$ Lifetime$(S2) = \emptyset$
$\Rightarrow$ nodes associated to `S1` and `S2` could be merged
pb: it increases the graph degree ...

lifetime splitting:

long lifetime increases the graph degree
$\Rightarrow$ split it into several parts ...
pb: where to split ?

Instruction scheduling

Motivation: exploit the instruction parallelism provided in many target architectures (e.g., VLIW processors, instruction pipeline, etc.)

Pbs:

- possible data dependancies between consecutive instructions (e.g., `x := 3 ; y := x+1`)
- possible resource conflicts between consecutive instructions (ALU, co-processors, bus, etc.)
- consecutive instructions may require various execution cycles
- etc.

$\Rightarrow$ Main technique: change the initial instruction sequence (instruction scheduling)

- preserve the initial pgm semantics
- better exploit the hardware resources

Rks: “loop unrolling” and “expression tree reduction” may help . . .

Dependency Graph

Data dependencies:

$\rightarrow$ execution order of 2 instructions should be preserved in the following situation:

Read After Write (RAW) : inst. 2 read a data written by inst. 1

Write After Read (WAR) : inst. 2 write a data read by inst. 1

Write After Write (WAW) : inst. 2 write a data written by inst. 1

Dependency graph G_D

- nodes = $\{$ instructions $\}$
- edges = $\{(i_1, d, i_2) \mid$ there is a dependency d from i_1 to $i_2\}$

Rk: if we consider a basic block, G_D is a directed acyclic graph.

Any topological sort of G_D leads to a valid result (w.r.t. pgm semantics).
This sort can be influenced by several factors:

- the resources used by the instruction ($\exists$ a static reservation table)
- the number of cycles it requires (latency)
- etc.

Example

1. Draw the dependency graph G_D associated to the following program
2. Give a topological sort of G_D
3. Rewrite this program with a “maximal” parallelism

```
1. a := x+1
2. x := 2+y
3. y := z+1
4. t := a*b
5. v := a*c
6. v := 3+t
```

Software pipelining (overview . . .)

Idea: exploit the parallelism between instrutions of distinct loop iterations

```
for k in 1 .. N loop
   r := T[k] ;          - inst. A
   x := x + r ;         - inst. B
   T[k] := x ;          - inst. C
end loop
```

Assumptions: 3 cycles per instruction, 1 cycle delay when no dependencies

- Initial exec. sequence: A(1), B(1), C(1), A(2), B(2), C(2), . . . A(k), B(k), C(k)
  $\boxed{\Rightarrow \text{7 cycles / iteration}}$
- "Pipelined exec. sequence": A(1), A(2), A(3), B(1), B(2), B(3), C(1), C(2), C(3), . . .
  $\boxed{\Rightarrow \text{3 cycles / iteration !}}$

(real life) pbs:

- `N` not always divisible by the number of instruction in the loop body
  `for k in 1 to N-2 step 3 loop A(k) ; A(k+1) ; A(k+2) ...`
- high latency instruction in the loop body
- possible overhead when k is not "large enough"
- . . .

Code Generation

Overview

1. Introduction
2. The “M” Machine
3. Code generation for basic `while`
4. Extension 1: blocks and procedures
5. Extension 2: some OO features

Main issues for code generation

- input : (well-typed) source pgm AST
- output : machine level code

Expected properties for the output:

- compliance with the target machine
  instruction set, architecture, memory access, OS, . . .
- correctness of the generated code
  semantically equivalent to the source pgm
- optimality w.r.t. non-functional criteria
  execution time, memory size, energy comsumption, . . .

A pragmatic approach

AST

$\downarrow$

$\boxed{\text{intermediate code generation}}$

$\downarrow$

Intermediate Representation 1 $\rightleftarrows$ $\boxed{\text{optimization(s)}}$

$\downarrow$

$\ldots$

$\downarrow$

Intermediate Representation n $\rightleftarrows$ $\boxed{\text{optimization(s)}}$

$\downarrow$

$\boxed{\text{(final) code generation}}$

$\downarrow$

target machine code

Intermediate Representations

- Abstractions of a real target machine
  - generic code level instruction set
  - simple addressing modes
  - simple memory hierarchy

- Examples
  - a “stack machine”
  - a “register machine”
  - etc.

Rk: other intermediate representations are used in the optimization phases . . .

The "M" Machine

- Machine with (unlimited) registers Ri
  special registers: program counter `PC`, frame pointer `FP`, stack pointer `SP`, register `R0` (contains always 0)
- Instructions, addresses, and integers take 4 bytes in memory
- Address of variable `x` is `E - offx` where:
  - `E` = address of the environment definition of `x`
  - `offx` = offset of `x` within this environment
    (staticaly computed, stored in the symbol table)
- Addressing modes:
  `Ri`, `val` (immediate), `Ri +/- Rj`, `Ri +/- offset`
- usual arithmetic instructions `OPER`: `ADD`, `SUB`, `AND`, etc.
- usual (conditional) branch instructions `BRANCH`: `BA`, `BEQ`, `BGT`, etc.

Instruction Set

instruction	informal semantics
`OPER Ri, Rj, Rk`	Ri $\leftarrow$ Rj oper Rk
`OPER Ri, Rk, val`	Ri $\leftarrow$ Rj oper val
`CMP Ri, Rj`	Ri-Rj (set cond flags)
`LD Ri, [adr]`	Ri $\leftarrow$ Mem[adr]
`ST Ri, [adr]`	Mem[adr] $\leftarrow$ Ri
`BRANCH label`	if cond then PC $\leftarrow$ `label`
	else PC $\leftarrow$ PC + 4
`CALL label`	branch to the procedure
	labelled with `label`
`RET`	end of procedure

The `while` *language*

$$
\begin{array}{lcl}
p & ::= & d \,;\, c \\
d & ::= & \text{var } x \mid d \,;\, d \\
s & ::= & x := a \mid s \,;\, s \mid \text{if } b \text{ then } s \text{ else } s \mid \text{while } b\ s \\
a & ::= & n \mid x \mid a + a \mid a * a \mid \ldots \\
b & ::= & a = a \mid b \text{ and } b \mid \text{not } b \mid \ldots
\end{array}
$$

Rk: terms are well-typed

$\rightarrow$ distinction between boolean and arithmetic expr.

Exo: Give the “M Machine” code for the following terms:

1. `y := x+42 * (3+y)`
2. `if (not x=1) then x := x+1`
   `else x := x-1 ; y := x ;`

Functions for Code Generation

$\texttt{GCStm} : \mathsf{Stm} \rightarrow \mathsf{Code}^*$

`GCStm(s)` *computes the code* `C` *corresponding to statement* `s`.

$\texttt{GCAExp} : \mathsf{Exp} \rightarrow \mathsf{Code}^* \times \texttt{Reg}$

`GCAExp(e)` *returns a pair (*`C`, `i`*) where* `C` *is the code allowing to 1. compute the value of* `e`*, 2. store it in* `Ri`*.*

$\texttt{GCBExp} : \mathsf{BExp} \times \mathcal{L}\mathsf{abel} \times \mathcal{L}\mathsf{abel} \rightarrow \mathsf{Code}^*$

`GCBExp(b, ltrue, lfalse)` *produces code* `C` *allowing to compute the value of* `b` *and branch to label* `ltrue` *when this value is "true" and to* `lfalse` *otherwise.*

Auxilliary functions

`AllocRegister` : $\rightarrow$ `Reg`
allocate a new register `Ri`

`newLabel` : $\rightarrow$ `Labels`
produce a new `label`

`GetOffset` : Var $\rightarrow \mathbb{N}$
returns the offset
corresponding to the specified name

$||$ denotes concatenation for Code sequences.

GCStm

GCStm (x := e)	=	Let	(C,i)=GCAExp(e),
			k=GetOffset(x)
		in	C$\|$ ST Ri, [FP-k]
GCStm (c_1 ; c_2)	=	Let	C_1 = GCStm(c_1),
			C_2 = GCStm(c_2)
		in	C_1 $\|$ C_2

GCStm (2)

```
GCStm (while e c)  =  Let  lb=newLabel(),
                           ltrue=newLabel(),
                           lfalse=newLabel()
                      in   lb:||
                           GCBExp(e,ltrue,lfalse)||
                           ltrue:||
                           GCStm(c)||
                           BA lb||
                           lfalse:
```

GCStm (3)

```
GCStm(if e then c1 else c2) = Let lnext=newLabel(),
                                  ltrue=newLabel(),
                                  lfalse=newLabel()
                              in  GCBExp(e,ltrue,lfalse)||
                                  ltrue:
                                  GCStm(c1)||
                                  BA lnext ||
                                  lfalse:||
                                  GCStm(c2)||
                                  lnext:
```

GCAexp

GCAExp(x)	=	Let	i=AllocRegister()
			k=GetOffset(x)
		in	((LD Ri,[FP-k]),i)
GCAExp(n)	=	Let	i=AllocRegister()
		in	((ADD Ri,R0,n),i)
GCAExp(e_1 + e_2)	=	Let	(C_1,i_1)=GCAExp(e_1),
			(C_2,i_2)=GCAExp(e_2),
			k=AllocRegister()
		in	(($C_1 \| C_2 \|$ ADD Rk, Ri_1,Ri_2),k)

GCBexp

GCBExp (e_1 = e_2,ltrue,lfalse)	=	Let	(C_1,i_1)=GCAExp(e_1),
			(C_2,i_2)=GCAExp(e_2),
		in	$C_1 \parallel C_2 \parallel$
			CMP Ri_1, Ri_2
			BEQ ltrue
			BA lfalse
GCBExp (e_1 et e_2,ltrue,lfalse)	=	Let	l=newLabel()
		in	GCBExp(e_1,l,lfalse)$\parallel$
			l:$\parallel$
			GCBExp(e_2,ltrue,lfalse)
GCBExp(NOT e,ltrue,lfalse)	=		GCBExp(e,lfalse,ltrue)

Exercises

- code obtained for
  - `y := x+42 * (3+y)`
  - ```
    if (not x=1) then x := x+1
                 else x := x-1 ; y := x ;
    ```
- add new statements (e.g, `repeat`)
- add new operators (e.g, `b ? e1 : e2`)
    ```

Extension 1: blocks

Blocks

Syntax

$$\begin{array}{rcl} S & ::= & \cdots \mid \textbf{begin } D_V \; ; \; S \textbf{ end} \\ D_V & ::= & \textbf{var } x \mid D_V \; ; \; D_V \end{array}$$

Rk: variables are unitialized and assumed to be of type **Int**

Problems raised for code generation

$\rightarrow$ to preserve scoping rules:

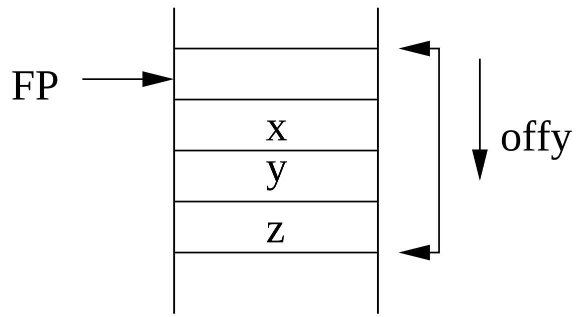
- local variables should be *visible* inside the block
- their *lifetime* should be limited to block execution

Possible locations to store local variables

$\rightarrow$ registers vs memory

Storing local variables in memory - Example 1

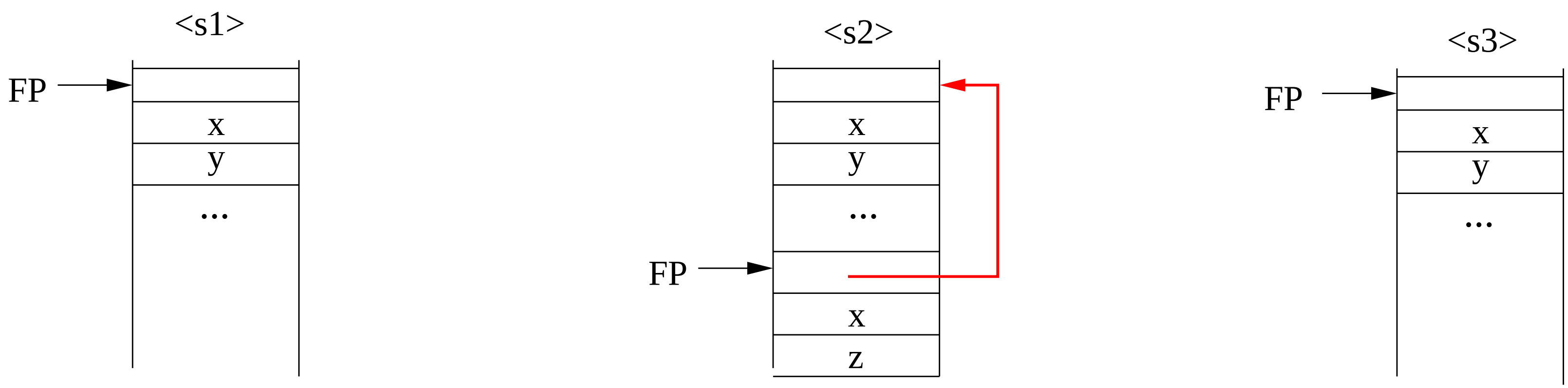
```
begin
   var x ; var y ; var z ;
   ...
end
```



- a *memory environment* is associated to each declaration Dv
- register FP contains the address of the current environment
- (static) offsets are associated to each local variables

Storing local variables in memory - Example 2

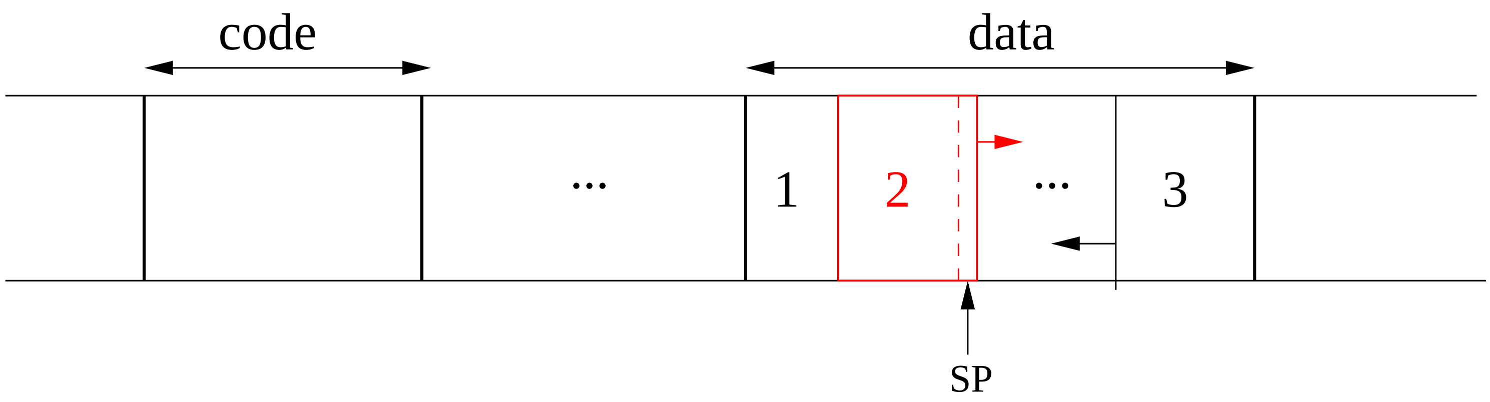
```
begin
   var x ; var y ; <s1>
   begin
      var x ; var z ; <s2>
   end ;
   <s3>
end
```



- entering/leaving a block $\rightarrow$ allocate/de-allocate a mem. env.
- nested block env. have to be linked together: "Ariane link"

$\Rightarrow$ a stack of memory environments . . . ($\sim$ operational semantics)

Structure of the memory



1: global variables

2: execution stack, SP = last occupied address

3: heap (for dynamic allocation)

Code generation for variable declarations

`SizeDecl` : $D_V \rightarrow \mathbb{N}$

`SizeDecl(d)` *computes the size of declarations* `d`

`SizeDecl (var x)`	=	4	(x of type `Int`)	
`SizeDecl` (d_1 ; d_2)	=	`Let`		v_1 = `SizeDecl`(d_1),
				v_2 = `SizeDecl`(d_2)
		`in`		$v_1 + v_2$

Code Generation for blocks

$$
\begin{array}{llll}
\texttt{GCStm(begin d ; s ; end)} & = & \texttt{Let} & \text{size} = \texttt{SizeDecl}(d), \\
 & & & C = \texttt{GCStm}(s) \\
 & & \texttt{in} & \texttt{ADD}, SP, SP, -4 \parallel \\
 & & & \texttt{ST}\ FP, [SP] \parallel \\
 & & & \texttt{ADD}\ FP, SP, 0 \parallel \\
 & & & \texttt{ADD}\ SP, SP, \text{size} \parallel \\
 & & & C \parallel \\
 & & & \texttt{ADD}\ SP, FP, 0 \parallel \\
 & & & \texttt{LD}\ FP, [SP] \parallel \\
 & & & \texttt{ADD}\ SP, SP, 4 \parallel
\end{array}
$$

With the help of some auxilliary functions . . .

`prologue(size)`	`epilogue`	push register(Ri)

`ADD SP, SP, -4`
`ST FP, [SP]`
`ADD FP, SP, 0`
`ADD SP, SP, size`
	`ADD SP, FP, 0`
	`LD FP, [SP]`
	`ADD SP, SP, +4`
		`ADD SP, SP, -4`
		`ST Ri, [SP]`

`GCStm` (`begin d ; s ; end`)	=	`Let`	size =`SizeDecl`(d),
			C=`GCStm`(s)
		`in`	Prologue(size) $\|\|$
			C $\|\|$
			Epilogue

Access to variables from a block ?

```
...
begin
   var ...
   x := ...
end
```

What is the memory address of x ?

- if x is a local variable (w.r.t the current block)
  $\Rightarrow$ adr(x) = FP + GetOffset(x)
- if x is a non local variable
  $\Rightarrow$ it is defined in a “nesting” memory env. E
  $\Rightarrow$ adr(x) = adr(E) + GetOffset(x)
  adr(E) can be accessed through the “Ariane link” ...

Access to non local *variables*

The number n of indirections to perform on the “Ariane link” depends on the “distance” between:

- the nesting level of the current block : p
- the nesting level of the target environment : r

More precisely:

- $r \leq p$
- $n = p - r$

$\Rightarrow$ n can be staticaly computed ...

Example

```
begin
   var x ; /* env. E1, nesting level = 1 */
   begin
     var y ; /* env. E2, nesting level = 2 */
     begin
        var z ; /* env. E3, nesting level = 3 */
        x := y + z /* s, nesting level = 3 */
     end
   end
end
```

From statement s:

- no indirection to access to z
- 1 indirection to access to y
- 2 indirections to access to x

Code generation for variable access

1. the nesting level r of each identifier x is computed during type-checking;
2. it is associated to each occurrence of x in the AST (via the symbol table)
3. function GCStm keeps track of the current nesting level p (incremented/decremented at each block entry/exit)

adr(x) is obtained by executing the following code:

- if $r = p$:

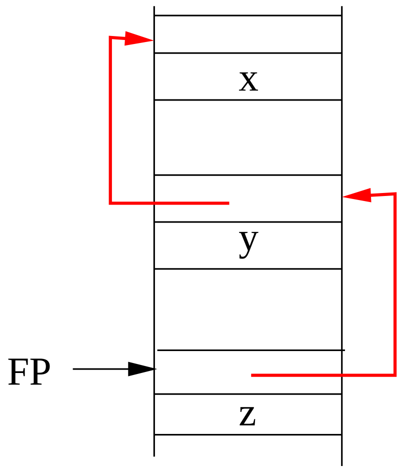
  FP + GetOffset(x)

- if $r < p$:

```
LD Ri, [FP]
LD Ri, [Ri]}   (p − r − 1) times
Ri + GetOffset(x)
```

Example (ctn'd)

```
begin
   var x ; /* env. E1, nesting level = 1 */
   begin
     var y ; /* env. E2, nesting level = 2 */
     begin
        var z ; /* env. E3, nesting level = 3 */
        x := y + z /* s, nesting level = 3 */
     end
   end
end
```




```
LD  R1, [FP]          ! R1 = adr(E2)
LD R2,  [R1 + offy]   ! R2 = y
LD R3,  [FP + offz]   ! R3 = z
ADD R4, R2, R3        ! R4 = y+z
LD  R5, [FP]
LD  R5, [R5]          ! R5 = adr(E1)
ST R4,  [R5 + offx]   ! x = y + z
```

Code generated for statement s

Extension 2: Procedures

Syntax

Procedure declarations:

$$\begin{array}{rcl} D_P & ::= & \textbf{proc}\; p\; (FP_L)\;\; \textbf{is}\; S\; ;\;\; D_P \mid \epsilon \\ FP_L & ::= & \textbf{x},\;\; FP_L \mid \epsilon \end{array}$$

Statements:

$$\begin{array}{rcl} S & ::= & \cdots \mid \textbf{begin}\; D_V\; ; D_P\; ;\;\; S\; \textbf{end} \mid \textbf{call}\; p(EP_L) \\ EP_L & ::= & AExp,\;\; EP_L \mid \epsilon \end{array}$$

FP_L: formal parameters list ; EP_L: effective parameters list

Rk: we assume here value-passing of integer parameters . . .

Example

```
var z ;

proc p1 () is
    begin
      proc p2(x, y) is z := x + y ;
      z := 0 ;
      call p2(z+1, 3) ;
    end

proc p3 (x) is
    begin
       var z ;
       call p1() ; z := z+x ;
    end

call p3(42) ;
```

Main issues for code generation

Procedure `P` is calling procedure `Q` . . .

Before the call:

- set up the memory environment of `Q`
- evaluate and “transmit” the effective parameters
- switch to the memory environment of `Q`
- branch to first intruction of `Q`

During the call:

- access to local/non local procedures and variables
- access to parameter values

After the call:

- switch back to the memory environment of `P`
- resume execution to the `P` instruction following the call

Access to non-local variables

```
proc main is
begin                /* definition env. of p */
   var x ;
   proc p() is x:=3 ;
   proc q() is
     begin
         var x ;
         proc r() is call p() ;
         call r() ;
     end ;
   call q() ;
end
```

Static binding $\Rightarrow$ when p is executed:

- acces to the memory env. of `main` =
  definition environment of the callee, static link
- acces to the memory env. of `r`
  memory environment of the caller, dynamic link

Information exchanged between callers *and* callees *?*

- parameter values
- return address
- address of the caller memory environment (dynamic link)
- address of the callee environment definition (static link)

This information should be stored in a memory zone:

- dynamically allocated

  (exact number of procedure calls cannot be foreseen at compile time)
- accessible from both parties

  (those address could be computed by the caller and the callee)

$\Rightarrow$

inside the execution stack, at well defined offsets w.r.t FP

A possible “protocol” between the two parties

Before the call, the caller:

- evaluates the effective parameters
- pushes their values
- pushes the static link of the callee
- pushes the return address, and branch to the callee’s 1st instruction

when it begins, the callee:

- pushes FP (dynamic link)
- assigns SP to FP (memory env. address)
- allocates its local variables on the stack

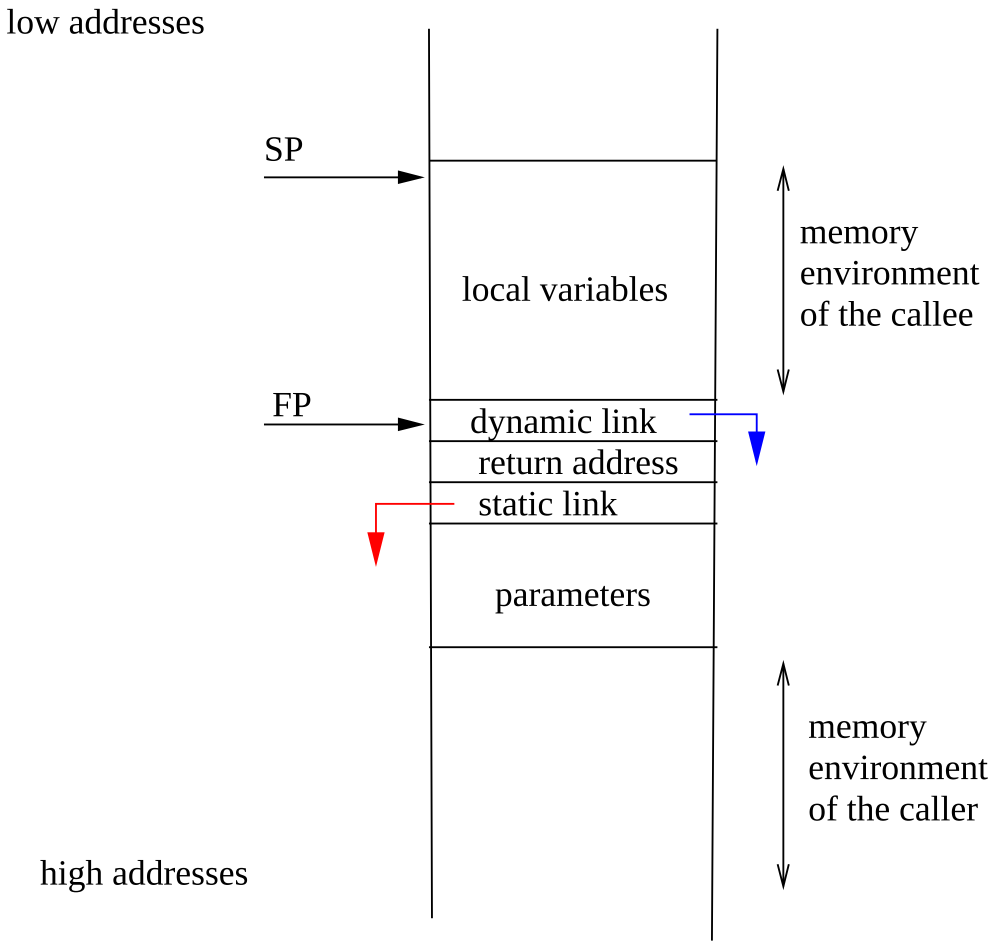
when it ends, the callee:

- de-allocates its local variables
- restores FP to caller’s memory env. (dynamic link)
- branch to the return address, and pops it from the stack

After the call, the caller

- de-allocates the static link and parameters

Organization of the execution stack



Addresses, from the callee:

loc. variables: FP+d, d<0

dynamic link: FP

return address: FP+4

static link: FP+8

parameters: FP+d, d>=12

Memory environment of the callee

...	0
Loc. var$_n$	$\leftarrow$SP, FP- 4*n
...	
Loc. var$_1$	$\leftarrow$FP
Dynamic link	$\leftarrow$FP
Return address	$\leftarrow$FP+4
Static link	$\leftarrow$FP+8
Param$_n$	$\leftarrow$FP+12
...	
Param$_1$	$\leftarrow$FP+8+4*n

Code generation for a procedure declaration

`GCProc` : $D_P \rightarrow$ Code*

`GCStm(dp)` *computes the code* `C` *corresponding to procedure declaration* `dp`.

`GCProc` (**proc** `p` (FP_L) **is** s end)	=	`Let`		
			C=`GCStm`(s)	
		`in`	Prologue(0) $\|$	
			C $\|$	
			Epilogue	

`GCProc` (**proc** `p` (FP_L) **is** begin dv ; dp ; s end)	=	`Let`	size =`SizeDecl`(dv),
			C=`GCStm`(s)
		`in`	Prologue(size) $\|$
			C $\|$
			Epilogue

Rk: this function is applied to each procedure declaration

Prologue & Epilogue

Prologue (size):

```
push (FP)              ! dynamic link
ADD FP, SP, 0          ! FP := SP
ADD SP, SP, -size       ! loc. variables allocation
```

Epilogue:

```
ADD SP, FP, 0          ! SP := FP, loc. var. de-allocation
LD FP, [SP]            ! restore FP
ADD SP, SP, +4
RET                    ! return to caller
```

RET:

```
LD PC, [SP]  //  ADD SP, SP, +4
```

Code Generation for a procedure call

Four steps:

1. evaluate and push each effective parameter
2. push the static link of the callee
3. push the return address and branch to the callee
4. de-allocate the parameter zone

```
GCStm (call p (ep))   =   Let   (C, size) = GCParam(ep)
                          in
                                C ||
                                Push (StaticLink(p)) ||
                                CALL p ||
                                ADD SP, SP, size+4
```

`CALL p`:

`ADD R1, PC, +4 // Push (R1) // BA p`

Parameters evaluation

`GCParam` : $EP_L \rightarrow$ Code$^* \times \mathbb{N}$

`GCStm(ep)=(c,n)` *where* `c` *is the code to evaluate and "push" each effective parameter of* `ep` *and* `n` *is the size of pushed data.*

`GCParam` (ε)	=	(ε, 0)
`GCParam` (`a ; ep`)	=	`Let`
		(Ca, i) = `GCAexp` (a),
		(C, size) = `GCParam` (ep)
		`in`
		(Ca $\parallel$ Push (R_i) $\parallel$ C, 4 + size)

Static link and non local variable access ?

- A global (unique) name is given to each identifier:

```
proc Main is
   proc P1 (...) is
     ...
         proc Pn (...) is
          begin
             var x ...
          end
```

$\rightarrow$ x is named $Main.P_1. \cdots .P_n.x$

- This notation induces a partial order:

$$(Main.P_1 \cdots .P_n \leq Main.P'_1 \cdots .P'_{n'}) \Leftrightarrow (n \leq n' \text{ and } \forall k \leq n.P_k = P'_k)$$

- For an identifier $x = Main.P_1 \cdots .P_n.x$,

  $x^\bullet = Main.P_1 \cdots .P_n$ is the definition environment of x

- For any identifier x (variable or procedure), procedure P can access x iff $x^\bullet \leq P$.

Examples

- A variable x declared in P can be accessed from P since $x^\bullet = P$ (hence $x^\bullet \leq P$).
- If g and x are declared in f, then x can be accessed from g since $x^\bullet = f$ and $f \leq g$.
- If x and f_1 are declared in $Main$, f_2 is declared in f_1, then x can be accessed from f_2 since $x^\bullet = Main$, $f_2 = Main.f_1.f_2$ ($x^\bullet \leq f_2$)
- If p_1 and p_2 are both declared in $Main$, x is declared in p_1, then x cannot be accessed from p_2, since $x^\bullet = Main.p_1$ and $Main.p_1 \not\leq Main.p_2$

Code Generation for accessing (non-) local identifiers

d_x: offset of x (variables or parameters) in its definition environment ($x^\bullet$)
P: current procedure

Condition	x = variable or parameter	x = procedure
$x^\bullet = P$	adr(x) = FP+d$_x$	SL(x) = FP
$x^\bullet < P$ $x = M.P_1 \cdots P_k$ $P = M.P_1 \cdots P_k \cdots P_n$	n-k-1 indirections `LD R,[FP+8]` `LD R,[R+8]`$\} \times (n-k-1)$ adr(x) = R+d$_x$	n-k-1 indirections `LD R,[FP+8]` `LD R,[R+8]`$\} \times (n-k-1)$ SL(x)=R

Back to the 1st example

```
var z ;

proc p1 () is
    begin
      proc p2(x, y) is z := x + y ;
      z := 0 ;
      call p2(z+1, 3) ;
    end

proc p3 (x) is
    begin
       var z ;
       call p1() ; z := z+x ;
    end

call p3(42) ;
```

Exercice:

- give the execution stack when p2 is executed
- give the code for procedures p1 and p2

Exercice

Consider the following extensions

- functions
- other parameter modes (by reference, by result)
- dynamic binding for variables and procedures ?

Procedures used as variables or parameters

```
var z1 ;
var p proc (int) ; /* p is a procedure variable */
proc p1 (x : int) is z1 := x ;
proc p2 (q : proc (int)) is call q(2) ;

proc q1 is
     begin
       var z1 ;
       proc q2 (y int) is z1 := x ;
       p := q2 ;
       call p ;
     end

p := p1 ;
call p ;
call p2 (p1) ;
```

Q: what code to produce for `p := ...` ? for `call p2(p1)` ? for `call p` ?

Information associated to a procedure at code level

```
p := q2
...
call p
```

To translate a procedure call, we need:

- the address of its 1st instruction
- the address of its environment definition

$\Rightarrow$ Variable p should store both information
$\Rightarrow$ At code level, a procedure type is a pair

(address of code, address of memory environment)

Exercice: code produced for the previous example ?