Index Coding : A means for efficient broadcasting

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Information Flow Problems

- "Information flow on a network" is a common framework for studying the transmission of information via a network.
- Information flow problems on a network are in general very hard problems to solve.
- **Index Coding** models a core problem of information flow problems.

Index Coding

The Index Coding Problem is an example of efficient broadcasting in the presence of side-information. It consists of the following.

- *One source* which has *messages* intended for the sinks.
- *Multiple sinks* (receivers) which demand some messages transmitted by the source
- Each sink possesses some prior knowledge of the source messages, called *side-information*.

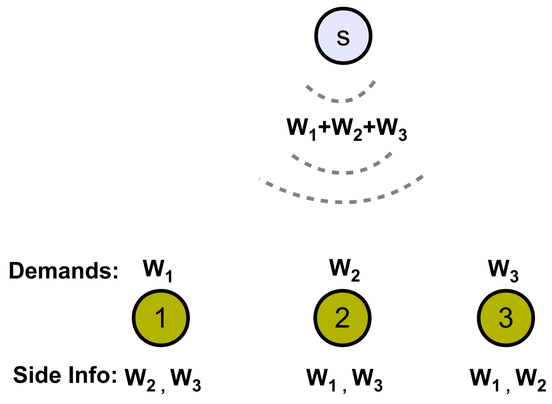

Figure 1: Index Coding Example

- A source has messages W_1, W_2, W_3, demanded by the three sinks respectively.
- Because of the side-information present at the sinks, it is sufficient to transmit just one symbol $W_1 + W_2 + W_3$ for all sinks to decode.

Rate of Index Coding

- The **Rate of Index Coding** is the ratio of the size of each message to the number of times the broadcast channel is used. We want to maximize this.
- Approaches to Index Coding are via three major methodologies - **Graph Theory, Interference Alignment, Random Coding.**
- A number of graphs can be associated to a given index coding problem, like the *side-information graph*, the *interference or conflict graph*, the *confusion graph*, *alignment graph*, and their properties can be used to characterise the index coding rates.

Main Objective of Index Coding

- Characterise Index Coding Problems which facilitate a certain rate of transmission.
- Design algorithms for obtaining transmission schemes (called an *Index Code*) which have maximum rate (which translates to minimal usage of the broadcast channel).

A Linear Algebra Framework for Index Coding

- In a ***linear index code***, the source transmits a vector codeword of the form,

$$\sum_i V_i W_i,$$

where W_i are the messages (as vectors of length LR), and V_i are ***pre-coding matrices*** (chosen at the source) of size $L \times LR$.

- ***Key observation:*** For any particular message k to be decoded at any sink j, the columns of V_k must be linearly independent of the space spanned by the (columns of the) precoding matrices of messages which are unavailable at sink j.
- The ***rate of the index code*** is then given as

$$R = \frac{\text{Length of the message vector} = LR}{\text{Length of the codeword} = L}.$$

- The maximum achievable rate of a given index coding problem can be characterised using the ***side-information graph***, which captures the interference structure of the IC problem.
- Recent progress resulted in complete characterisation of rate $\frac{1}{2}$ and partial characterisation of rate $\frac{1}{3}$ (Reference [[1]])

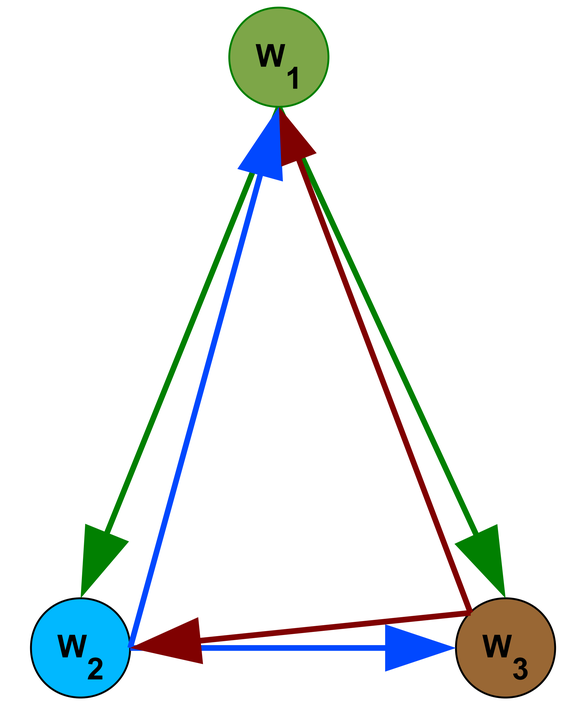

Figure 2: Side-information graph of Example. Edges indicate the side-information available at the sinks which demand the corresponding messages.

Areas related to Index Coding

- Index Coding forms a core problem of several classes of information flow problems.
- Examples include -
- **Network Coding** (flow of information in a network).
- **Coded Caching** (Caching coded packets of information by network users).
- **Locally Repairable Codes** (Error Correcting Codes for Big Data)

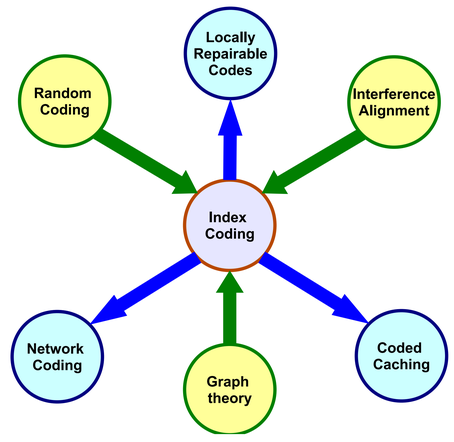

Figure 3: Approaches to Index Coding and Areas influenced by Index Coding

References

[1] [Recent] Prasad Krishnan and V. Lalitha, "A class of index coding problems with rate $\frac{1}{3}$", Jan. 2016, Available on arXiv at http://arxiv.org/pdf/1601.06689.

[2] Z.Bar-Yossef, Y. Birk, T.S. Jayram, T. Kol, "Index Coding with Side Information", IEEE Transactions on Information Theory, Vol. 57, No. 3, March 2011, pp. 1479-1494.

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