A class of index coding problems with rate $\frac{1}{3}$

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Outline

Groupcast Index Coding from the Interference Alignment perspective

Known results for groupcast

Our result for rate $1/3$ index codes

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Groupcast index coding

- A broadcast channel between the source and T receivers
- Messages $\mathcal{W} = \{W_i \in \mathbb{F}, i \in [1:n]\}$.
- Demand set at receiver j : $D(j) \subseteq \mathcal{W}$
- Side information at receiver j: $S(j) \subseteq \mathcal{W} \backslash D(j)$

Linear Index Code and its Rate

- Index code:
  Map from $\{$Messages$\} \rightarrow \{$L-length codewords$\}$
- Rate $R = \frac{1}{L}$.
- Our results can be generalised to vector-linear index codes.

Interference alignment framework for index codes

- Index Coding map be $B_{L \times n}$; Transmitted vector is $B\boldsymbol{W}$.
- Consider a sink j which demands message W_k.
- Sink j can cancel the contributions from $S(j)$, obtaining

$$\sum_{i:i \notin S(j)} W_i \boldsymbol{b_i} = W_k \boldsymbol{b_k} + \sum_{i:i \notin S(j) \cup \{k\}} W_i \boldsymbol{b_i}$$

- Let $I(j,k) = \{W_i : i \notin S(j) \cup \{W_k\}\}$.
- Decoding is possible if $\boldsymbol{b_k}$ is independent of space spanned by vectors assigned to $I(j,k)$ (interference constraints).
- Choose matrix B such that this is satisfied with least possible L (alignment opportunities).

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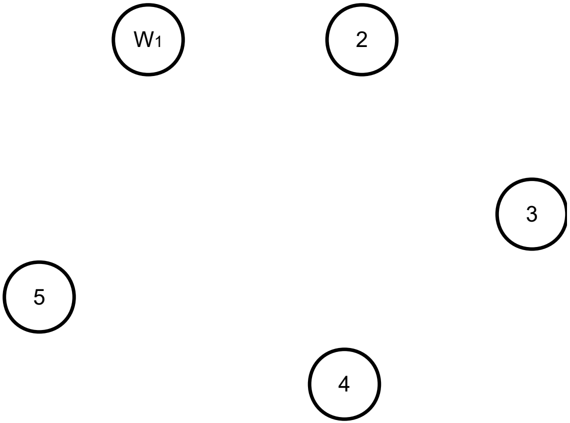
- Tehrani et al, ISIT 2012, “Bipartite Index Coding”.
- Maleki et al, “Index Coding - An Interference Alignment Perspective”, ISIT 2012, TIT Sep. 2014.
- Jafar, “Topological Interference Management Through Index Coding”, IEEE TIT, Jan 2014 [Jaf].

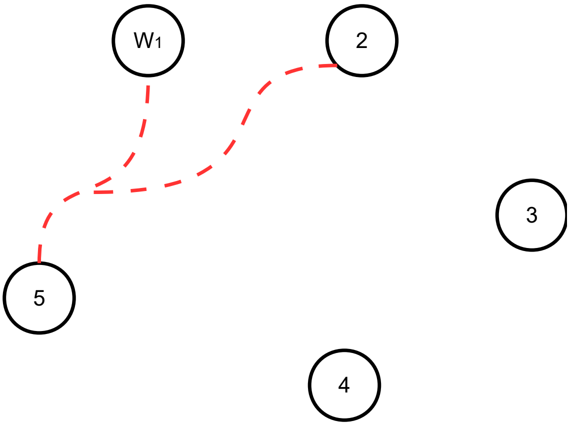
Known results for groupcast

- **Rate 1**: Each receiver demands exactly one message, and has all others as side-information.
- **Rate $\frac{1}{2}$**: [Jaf] via *alignment* and *conflict* graphs.

Alignment graph and conflict hypergraphs



Alignment graph and conflict hypergraphs



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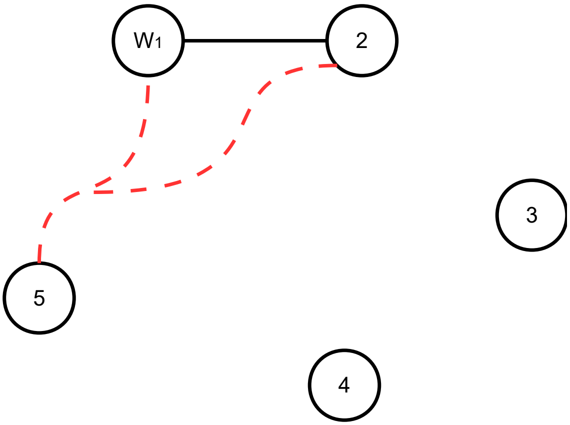

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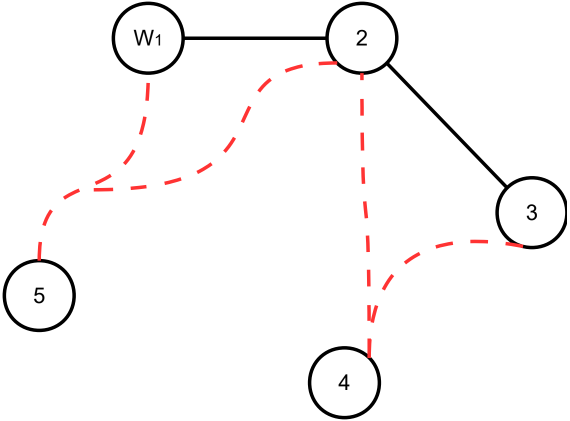
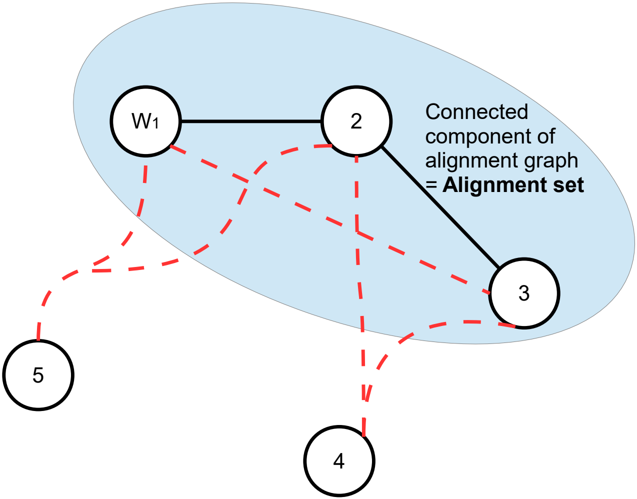


Alignment sets



Rate $\frac{1}{2}$ result from [Jaf]

Theorem (Rate $\frac{1}{2}$)

An index coding problem is rate $\frac{1}{2}$ feasible if and only if there are no internal conflicts (conflicts within alignment sets).

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Groupcast Index Coding from the Interference Alignment perspective

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Our contribution

Main result (A rate $\frac{1}{3}$ feasible class of index coding problems)

A rate $\frac{1}{2}$ infeasible IC problem is rate $\frac{1}{3}$ feasible if every alignment set satisfies one of the following properties

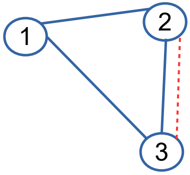
- It doesn't have both forks and cycles (follows from [Jaf]).
- It is a *type-2 alignment set* with no *restricted internal conflicts*.

Theorem (Known from [Jaf])

A rate $\frac{1}{2}$ infeasible IC problem is rate $\frac{1}{3}$ feasible if no alignment set has both forks and cycles (i.e., $|I(j,k)| \leq 3, \forall j,k$).

Type-2 alignment set

- **Triangular interfering set:** A set of three messages interfering at some receiver, with at least two of them in conflict
- Any two triangular interferers are ‘adjacent’ if they are meeting at conflict edges.
- **Type-2 alignment set:** A ‘connected component’ of such triangular interfering sets.



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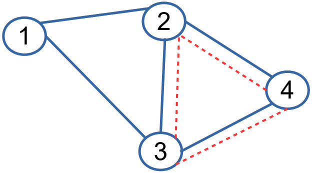

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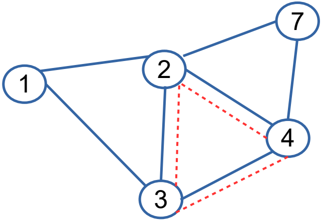

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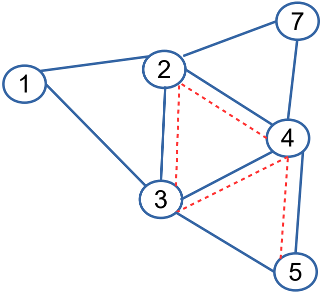

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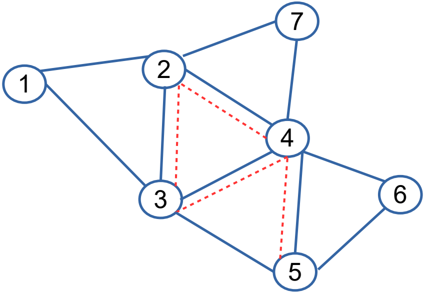

The condition on the type-2 alignment set

Type-2 alignment set with no *restricted internal conflicts*.

$\Updownarrow$

IC problem restricted to Type-2 alignment set is rate $\frac{1}{2}$ feasible.

$\Updownarrow$

Type-2 alignment set can be assigned vectors from a two dimensional space with all its internal conflicts resolved.

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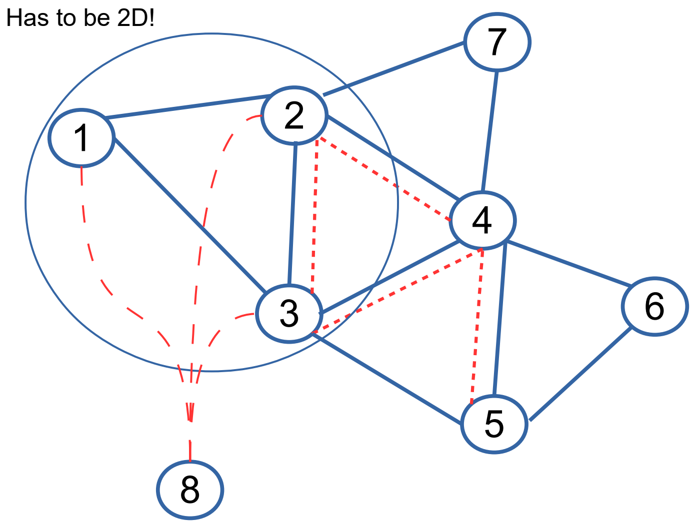
$\longrightarrow$ A **necessary** condition for rate $\frac{1}{3}$ feasibility

A necessary condition for rate $\frac{1}{3}$

Theorem A

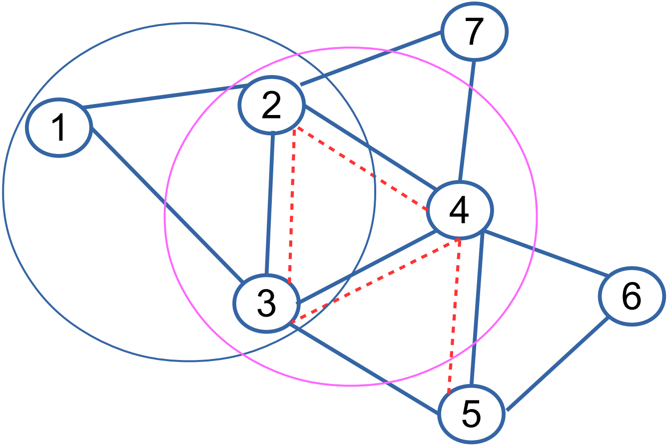
An IC problem is rate $\frac{1}{3}$ feasible only if any type-2 alignment can be allocated vectors from a two dimensional vector space with all its internal conflicts resolved.

Proof of the Necessary Condition for rate $\frac{1}{3}$



Proof of the Necessary Condition for rate $\frac{1}{3}$

Intersection has to be 2D



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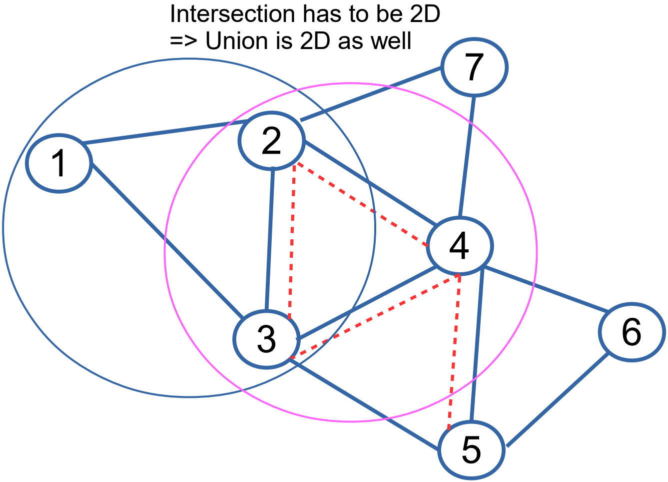

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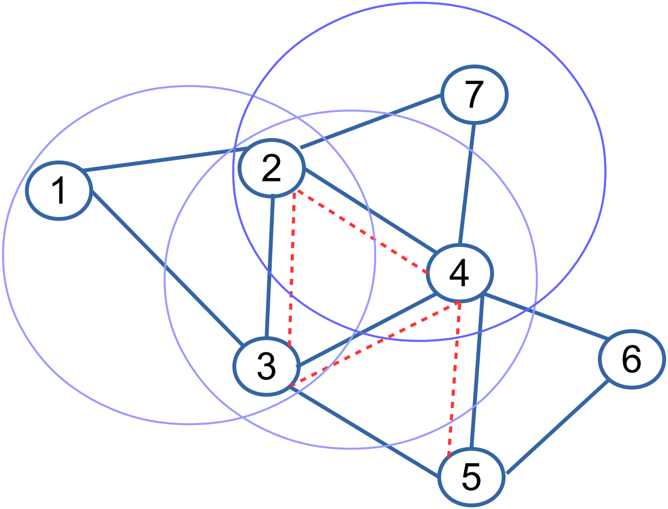

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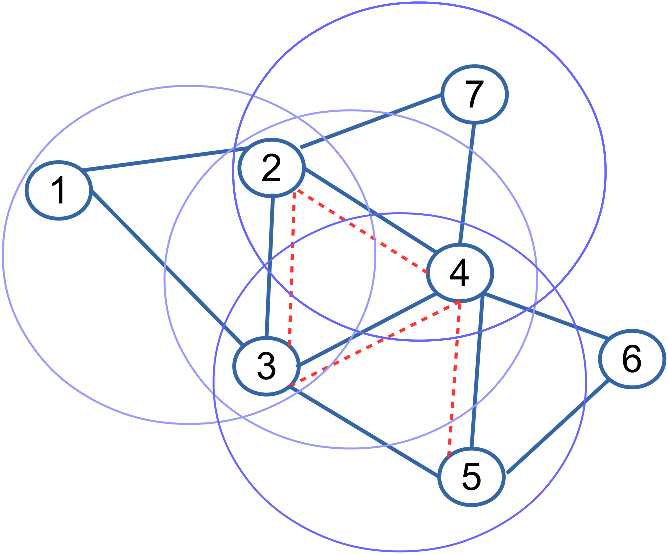
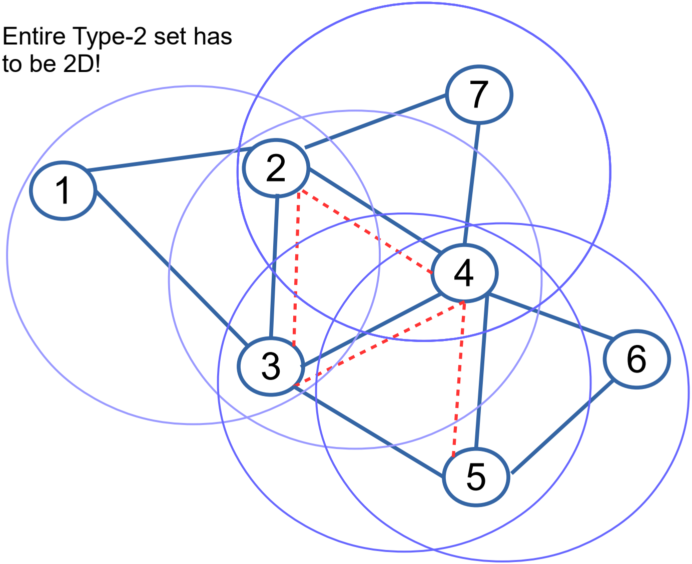


Proof of the Necessary Condition for rate $\frac{1}{3}$

Entire Type-2 set has
to be 2D!



The condition on the type-2 alignment set

Type-2 alignment set with no *restricted internal conflicts*.

$\Updownarrow$

IC problem restricted to Type-2 alignment set is rate $\frac{1}{2}$ feasible

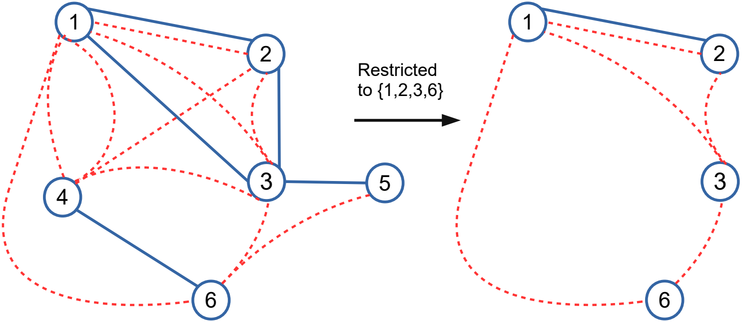
$\Updownarrow$

Type-2 alignment set can be assigned vectors from a two dimensional space with all its internal conflicts resolved.

$\longrightarrow$ A necessary condition for rate $\frac{1}{3}$ feasibility : ✓

Restricted IC problem and restricted conflicts

- For $\mathcal{W}' \subset \mathcal{W}$, the **IC problem restricted to $\mathcal{W}'$** considers all demands and side-information only within $\mathcal{W}'$ at receivers.
- Restricted alignment graphs, Restricted conflict graphs.
- **Restricted internal conflicts:** Conflicts within restricted alignment sets.



The condition on the type-2 alignment set

Type-2 alignment set can be assigned vectors from a two dimensional space with all its internal conflicts resolved.

$\Updownarrow$ **(Projection to $\mathbb{F}^2$)**

IC problem restricted to type-2 alignment set is rate $\frac{1}{2}$ feasible

$\Updownarrow$ **(Rephrasing)**

Type-2 alignment set with no *restricted internal conflicts*.

How to assign vectors if the conditions in Main Result are met?

- All alignment sets are one of three types. Assign vectors differently in each case.

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- *Alignment set which has no three messages interfering at any receiver:* Assign a random 3×1 vector to each message.
- *Alignment set which consists only of three messages interfering at any receiver without any conflicts in-between:* Assign the same random vector to all messages.
- *Alignment set which is a type-2 alignment set without restricted internal conflicts:* For each restricted alignment set, assign one randomly generated vector from a 2D space.

Open problems

- Solving the rate $\frac{1}{3}$ problem.
- Generalize to other rates.

Thank you!