

A T-Step Ahead Constrained Optimal Target Detection Algorithm for a Multi Sensor Surveillance System

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Abstract – We present a methodology for optimal target detection in a multi sensor surveillance system. The system consists of mobile sensors that guard a rectangular surveillance zone crisscrossed by moving targets. Targets penetrate the surveillance zone with poisson rates at uniform velocities. Under these conditions we present a motion strategy computation for each sensor such that it maximizes target detection for the next T time-steps. A coordination mechanism among sensors ensures that overlapping and overlooked regions of observation among sensors are minimized. This coordination mechanism is interleaved with the motion strategy computation to reduce detections of the same target by more than one sensor for the same time-step. To avoid an exhaustive search in the joint space of all the sensors the coordination mechanism constrains the search by assigning priorities to the sensors and thereby arbitrating among sensory tasks. A comparison of this methodology with other multi target tracking schemes verifies its efficacy in maximizing detections. “Sample” and “time-step” are used equivalently and interchangeably in this paper.

Index Terms- multi sensor system, optimal target detection, sensor network, sensor surveillance, multi-agent system

I INTRODUCTION

We present a framework for maximizing target detections in a surveillance system consisting of multiple mobile sensors (i.e., robots). The sensors collectively guard a rectangular surveillance zone crisscrossed by moving targets. A-priori information about targets is the knowledge of their poisson rates of entry. Current knowledge about all targets within the sensing range of a sensor is available to the sensor in the form of the target’s velocity information and motion direction. While current information is used in deterministically computing the best places to be at in the subsequent time steps, it does not take into account the changes in the environment that occur due to consistent flow of target traffic into and out of the surveillance zone. Knowledge of the statistics of target entry and exit on the other hand are used to compute the statistically optimal places to be at in the future from a given location, but does not consider the current information gleaned through sensor observations. Hence a combination of both current target information and target statistics is used to compute a motion strategy of a sensor that maximizes the number of detections for the next T time steps.

Computing the best strategy from its individual perspective can result in motions for each sensor with overlapping regions of observation. A priority based

coordination mechanism is interleaved with the motion strategy computation to reduce detections of the same target by more than one sensor. To avoid an exhaustive search in the joint space of all the sensors the coordination mechanism constrains the search by assigning priorities to the sensors. Hence while optimal from a single sensor’s perspective the algorithm is not optimal across all sensors – a choice that one is reasonable to assume in light of prevailing real time demands. The details of this method are presented in section 3 of this paper. Extensive simulation results confirm the efficacy of this strategy in the form of tabular comparisons presented in section 4.

Multi-sensor surveillance finds applications such as in border patrol, guarding of secured areas, search and rescue and warehouse surveillance [1, 2]. It involves detection of multiple intrusions and/or tracking through coordination between the sensors. Detection and target tracking has been researched from multiple viewpoints. Some efforts have focused on the problem of identifying targets from a given set of data through particle filters [3], and probabilistic methods [4]. The problem of data association or assigning sensor measurements to the corresponding targets were tackled by Joint Probabilistic Data Association Filters by the same researchers such as in [3]. Kluge and others [5] use dynamic timestamps for tracking multiple targets. Krishna and Kalra [6] presented clustering based approaches for target detection and further extended it to tracking and avoidance. The focus of these approaches has been on building reliable estimators for predicting target trajectories that is different from the objective of this effort to maximize target detections.

In the context of distributed task allocation and sensor coordination Parker proposed a scheme for delegating and withdrawing robots to and from targets through the ALLIANCE architecture [7]. The protocol for allocation was one based on “impatience” of the robot towards a target while the withdrawal was based on “acquiescence”. Jung and Sukhatme [8] present a strategy for tracking multiple intruders through a distributed mobile sensor network and a strategy for maximizing sensor coverage[8, 9]. Lesser’s group have made significant advances to the area of distributed sensor networks [10] and sensor management [11]. In [12] Parker presents a scheme called A-CMOMMT where the goal is to maximize the number of targets observed over a time interval of length T based on the same philosophy of behavior-based control as in [7]. The authors of this paper present their scheme for resource allocation and coordination in a distributed sensor

system through a set of fuzzy rules in [13] and further compare various resource allocation strategies in terms of their detection performance in [14]. The author of [15] has looked at the problem of static placement of sensors in known polygonal environments and [16] describes a distributed sensor approach to target tracking using fixed sensor locations. The current approach is disparate from those of [15,16] in that in the current scheme the sensors are mobile.

Among the approaches that we have encountered the closest to this are [12] and [8]. In [12] a behavior-based approach, A-CMOMMT, is compared with three other heuristic approaches where the sensor's motion strategy is arbitrary or random in the first, stationary (the sensor does not move) in the second and based on local force control in the third. In [8] a motion strategy for tracking multiple targets based on density estimates is presented. The robot attempts to maximize target detections by maintaining itself at a particular distance from the center of gravity of currently observed targets. In these approaches since no assumption is made regarding target arrival statistics the motion strategy does not guarantee sensors move to best possible locations to optimize their detections.

In contrast, our approach presents a constrained optimal scheme that moves the sensor to regions in the surveillance region to maximize detections over the next T samples. The optimality of this approach comes with the tradeoff of apriori knowledge regarding target statistics. Nonetheless this situation is not new in robotic and multi-robotic literature where optimal path planning and scheduling algorithms require prior knowledge of the workspace in which they operate in terms of their static and dynamic contents vis-à-vis behavior based approaches that do not guarantee optimality or completeness but require no prior knowledge.

III. THE METHODOLOGY

A. Description of Surveillance Zone, Sensors and Targets:

We consider the surveillance system depicted in figure 1. The sides of the outer rectangle or the biggest rectangle in figure 1 form the boundary of the surveillance zone – the area enclosed by it is the area of interest where sensors attempt to optimize their rates of detection. The shaded circles are effective sensor ranges in their starting positions. The field of vision (FOV) of a sensor is 360 degrees. The squares with thick boundaries in figure 1 with the sensors at their center are the inscribed squares of the circular FOV of a sensor. In other words, the diameter of the FOV of a sensor is the diagonal of one such inscribed square. Purely from the point of view of facilitating easier computations the sensor considers only those targets that lie within its inscribed square as targets within its FOV. It needs to be emphasized that this simplification does not have any bearing on the overall philosophy of this approach. In the results section the efficacy of this method is verified by uniformly applying this same condition across all other approaches that are compared and the extension to a case of circular FOV is merely one of more

involved but computable computations. The entire surveillance region is discretized into a lattice of cells. The cells are represented as the small squares inside the FOV of the leftmost and topmost sensor. The dashed lines along the length and breadth indicate that the cells proceed to fill the entire surveillance zone. At each of the cell locations various aspects of target statistics are computed that are described later. The crosses outside the surveillance zone are the source points from where targets emanate as per Poisson statistics. Targets percolate from each of those sources into that horizontal or vertical half-plane that contains the surveillance zone. Therefore, all targets coming from a particular source will be contained within an angular span of π radians. Furthermore, the following assumptions are made for sensors and targets

- A sensor can detect all targets within its FOV or occlusion relations are not considered.
- The takeoff angle of a target from its source point is uniformly distributed in $[0, \pi]$
- All targets move with the same uniform velocity within the surveillance zone along linear trajectories, which can be ascertained by the sensor.

The last assumption allows that the statistical values of various parameters computed at every cell in the lattice to take a unique value rather than a probability distribution. In case of a distribution the expected values of the parameters need to be made use of.

B Problem Statement and Approach

The problem attacked in this paper is stated as follows. Given:

- N_S : The set of all sensors in the system, $N_S = \{s_0, s_1, \dots, s_{n_s}\}$, where s_i denotes the sensor with label i , ordered in a sequence. Hence n_s the label of the last enumerated sensor is also the number of sensors in the system, which is a constant.
- $N_{I,t}$: The set of all targets in the system at time t , where, $N_{I,t} = \{i_0, i_1, \dots, i_{n_{I,t}}\}$, and $n_{I,t}$ is the number of targets in the system at time step t that varies at every time step and hence dynamic.
- $g_{m,t}$: A binary variable that takes the value 1 if a target im is observed by any one of the n_s sensors at t .

Objective: To develop an algorithm such that the

following cost function $J = \sum_{t=1}^T \sum_{m=1}^{n_{I,t}} g_{m,t}$ is maximized. In

other words, the number of detections of the targets present in the system at every time step is maximized over T time steps.

Approach:

1) At each cell, P_i , in the lattice the following are computed.

1a). $\hat{\lambda}_{P_i}$: The expected rate of target entry into the FOV of a sensor centered at P_i . If there are Q target sources each

Step 1 of the algorithm is performed only once at the start of the simulation across all the cells and is essentially an offline step unless the target statistics changes dynamically.

B Computing $\hat{n}_d^{k,t}$

Computation of $\hat{n}_d^{k,t}$ varies if it is being carried out for the same cell location for a future time or for a different cell location. For the case of computing at the same cell the procedure is as follows:

Given that the sensor at cell P_i currently or at $t=0$ observes n_{P_i} number of targets then the number of targets it is likely to see T time-steps into the future is given by:

$$\hat{n}_d^{i,T} = n_{P_i} - k + \hat{\lambda}_{P_i}T - \sum_{j=1}^Q \hat{\lambda}_{P_i,j} \sum_{t=1}^T P(te_{P_i,j} < T-t) \quad (1)$$

Here $\hat{\lambda}_{P_i}$, $te_{P_i,j}$, $\lambda_{P_i,j}$ have the same connotations as discussed in step 1 and Q is the number of target sources as before. The first term on the right hand side or RHS of equation (1) is the deterministic part that is computed purely based on what the sensor senses now. It merely states that out of n_{P_i} particle seen currently k of them would disappear by T time steps. The second term onwards on the RHS denote the statistical counterpart. It says that $\hat{\lambda}_{P_i}T$ are likely to enter the FOV in T time steps and of which the number in the third term are likely to leave the FOV by T . The third term containing the summation is explained as follows. $P(te_{P_i,j} < T-t)$ denotes the probability that a target from source j that entered the FOV at t would have escaped the FOV by T . Hence $\hat{\lambda}_{P_i,j}P(te_{P_i,j} < T-t)$ is the estimate of the number of particles that entered at t and escaped by T , since $\hat{\lambda}_{P_i,j}$ is the estimate of number of particles entering for every t from j into the FOV of the sensor at P_i . The summation over t signifies that estimate of the number of particles that would have left by T needs to be done in correspondence with the time at which they entered between now and T . The summation over Q signifies that the pdf of $te_{P_i,j}$ is not the same for every source j at that cell location. The $P(te_{P_i,j} < T-t)$ is given below by its pdf with notations from step 1.

$$P(te_{P_i,j} < T-t) = \begin{cases} 0 & ; T-t < t_a \\ \frac{1}{\eta_{P_i,j}} \int_{t_a}^T f(te_{P_i,j}); t_a \leq T-t \leq t_b & ; t_a \leq T-t \leq t_b \\ 1 & ; T-t > t_b \end{cases}$$

The number of candidates likely to be seen for a different cell P_k at T given that n_{P_i} number of targets then the number of targets are currently seen at P_i is given by:

$$\hat{n}_d^{k,T} = n_{P_i} - k + \kappa \left(\hat{\lambda}_{P_i}T - \sum_{j=1}^Q \hat{\lambda}_{P_i,j} \sum_{t=1}^T P(te_{P_i,j} < T-t) \right) + (1-\kappa)\hat{d}_{P_k} \quad (2)$$

Equation (2) has similar connotations as (1) except for the appearance of κ that represents the fraction of area common to FOV erected at P_i and P_k and \hat{d}_{P_k} is as discussed in step 1. For that fraction of the FOV at P_k that is not visible from P_i , \hat{d}_{P_k} , which is the expected number of detections at cell P_k at any instant is made use of. And since $(1-\kappa)$ is the fraction of FOV at P_k that is not visible \hat{d}_{P_k} is multiplied by that fraction. This makes use of the assumption that the distribution of targets is uniform across an area. In our plots of \hat{d}_{P_k} done for the surveillance zone variations between cells are steep only towards the edges of the zone. A more rigorous computation involves evaluating the pdf for the overlapped and non-overlapped areas separately. This is being avoided since (2) is computed during the online phase of the algorithm. This assumption is also invoked in step 6 while reducing the number of detections due to overlapping FOV.

C The Coordination Phase:

The coordination phase prevents paths of sensors to come close to one another to avoid overlapping FOV. This is done by imposing penalties and reducing values of $\hat{n}_d^{k,t}$ and recomputing paths for sensors with lower priorities as described in steps 5, 6 and 7. The path of the sensor with highest detections is fixed. The path with the second highest recomputed if there are overlapping areas of observation with the first. The recomputed path constitutes a motion strategy that is optimal in terms of detections under the constraint that the path of highest sensor is fixed. Similarly the path of the sensor with the least priority sensor when recomputed is optimal under the constraint that the paths of the sensors with higher priority are fixed. Hence at the coordination phase the optimality of the algorithm is not complete. A fully complete algorithm would involve a search in the joint space of all sensors that is combinatorially hard..

IV SIMULATION RESULTS

This section reports results obtained through simulations on our environment developed using Borland's JBuilder IDE. The value of the number of time steps, T , used for these simulations is 3, carried out on a P4 workstation with clock speed of 1.8 GHz. Figure 3 shows a snapshot of the environment with 10 sensors. The current position of the sensor is shown through the bigger shaded circle, while the targets through smaller ones. The traces of sensor movements are also shown. Target traces are not shown to avoid cluttering.

Table 1 compares the performance of the system in the presence and absence of coordination. The experiments done with sensor coordination are labeled as 1a, 2a, ..., while those without coordination are labeled as 1b, 2b,.... Each experiment lasted for 150 time steps in total. The first column of the table denotes the index of the experiment; the second denotes the number of sensors in the system, the third the average detections or the number of targets detected by at-least one sensor per sample. The fourth signifies the average fraction of targets detected per sample, which is the number of targets detected per sample by one or more sensors divided by the number of targets in the surveillance zone per sample. Columns 5, 6 and 7 represent the number of targets detected exactly by one, two and three sensors per sample. The last column denotes the target velocity. Detections by more than three sensors are not tabulated for they generally assume insignificant values. Of particular interest in comparing the two schemes are the average fraction of target detections as well as the number of targets detected by exactly one, two and three sensors. It is expected that the average fraction of detections as well as the number of targets detected by exactly one sensor to be more for the method with coordination incorporated. Simultaneously the number of targets detected by two or more sensors is anticipated to be higher for the method that does not use the coordination phase. As the overlapping areas of FOV are higher in the absence of coordination the number of targets detected by two or more sensors is also higher. This in turn allows more targets to go undetected and the average fraction of detections to be lesser. The abbreviations used in the column headings are explained in the table caption.

EN	NS	AD	ZD	AF	D1S	D2S	D3S	TV
1a	10	32	22	0.6	29	2	0	10
1b	10	27.1	24.9	0.52	19.5	6.8	0.8	10
2a	6	22.2	25.7	0.46	22	0.2	0	10
2b	6	21.5	26.8	0.44	18.4	3.2	0	10
3a	10	20.4	21.1	0.49	20	0.3	0	15
3b	10	17.3	24.1	0.41	13.8	3.5	0	15
4a	6	27.2	20.2	0.57	26.5	0.7	0	10
4b	6	26.9	20.4	0.56	25.7	1.3	0	10
5a	10	20	22.5	0.47	19.6	0.4	0	15
5b	10	18.7	23.8	0.44	16.4	1.5	0.9	15

Table 1: Comparison between motion strategies with and without sensor coordination. Experiments with labels ending in ‘a’ or the first line in every row have coordination incorporated while those ending in ‘b’ or the second line in each row are uncoordinated. Abbreviations are as follows: EN = experiment number, NS = number of sensors, AD = Average Detections per sample, ZD = Zero Detections per sample, AF = Average Fraction per sample, D1S = Detections by exactly 1 sensor, D2S = Detections by exactly 2 sensors, D3S = Detections by exactly 3 sensors, TV = Targets’ Velocity

As seen from table 1 the performance of the coordinated scheme with decoupled optimization is better for all experiments with differing NS and TV values than the method sans coordination. For example the number of targets detected by exactly one sensor per sample, the D1S column, is significantly higher in a number of experiments for the coordinated vis-à-vis the uncoordinated scheme. It is higher by 10, 4 and 6 detections per sample in experiments 1, 2 and 3. Also the number of targets detected by two sensors is significantly lesser across all experiments for the coordinated method. In experiments 1 and 5 the number of targets detected by three sensors is almost one per sample for the method lacking coordination. Correspondingly average fraction of targets detected per sample for the coordinated method is higher in all the experiments. The results of the table tally with the expectations and the underlying reasons for these expectations mentioned earlier.

In table 2 we compare the current strategy with four of our previous methods of target detection and pursuit [14]. The first column signifies the experiments. Indexes 1a, 2a, etcetera correspond to the current method. Those with indices ‘b’, ‘c’, ‘d’ and ‘e’ correspond to coordinated-distracted, coordinated-dedicated, local-distracted and local-dedicated methods of resource allocation of sensors to targets for target detection [14]. Table 2 shows that across all experiments the current scheme surpasses all others in terms of average fraction of targets detected as well as other criteria such as D1S, D2S, AD and AF. It is to be noted that a difference of 0.06 in average fraction per sample between two methods corresponds to a difference of 4 or 5 target detections per sample (there are 70 to 80 targets in the zone per time-step on an average), which in turn corresponds to a difference of 600 detections in 150 time-steps of simulation. The value of λ was 0.3 and $Q=8$ in these simulations.

V DISCUSSIONS and COMMENTS

Sub-Optimality: The algorithm is made suboptimal merely to reduce the search space involved in making it

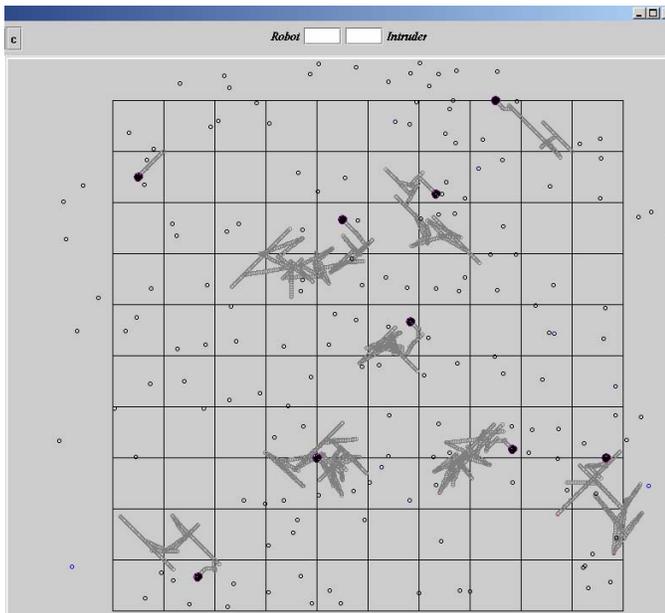


Figure 3: A simulation system with ten sensors and several targets. The robots are shown through larger circles and targets through smaller ones. The traces of sensors are also shown.

globally optimal. A straightforward implementation would involve a direct search in the space of all sensors by extending this approach. The method of assigning priorities to constrain the search has its counterpart in decoupled priority based approaches to multi robot path planning [17,18].

Generality and Assumptions Involved: The current approach retains its utility of optimality for any given arrival profile or arrival statistics and in that sense generalized in principle. The details vary with respect to how equations (1) and (2) are computed for different target statistics but the overall approach remains the same. It is not binding the algorithm that the velocities of the targets be constant and uniform. The algorithm shall continue to be optimal in a statistical sense as long as the velocity profile and direction profile is known or can be modeled. Varying the velocity of the targets leads to distributions of various parameters at each grid point rather than unique values that in turn makes the optimality of algorithm less deterministic and more statistical.

EN	NS	AD	ZD	AF	D1S	D2S	D3S	TV
1a	6	22.3	25.7	0.46	22	0.2	0	10
1b	6	20.3	27.7	0.42	17.9	2.3	0.1	10
1c	6	18.3	30.5	0.37	13.1	4.4	0.8	10
1d	6	21	27	0.43	18.7	2.3	0	10
1e	6	18.3	30.5	0.37	13.1	4.6	0.5	10
2a	10	21.5	20.2	0.46	21.0	0.5	0.0	15
2b	10	19.0	26.1	0.40	17.5	1.6	0.0	15
2c	10	19.7	25.8	0.42	18.0	1.6	0.02	15
2d	10	19.2	25.4	0.41	17.0	2.2	0.02	15
2e	10	20.0	25.8	0.42	18.8	1.2	0.0	15
3a	6	27.2	20.2	0.57	26.5	0.7	0	10
3b	6	21.2	26.1	0.44	20.2	1.0	0	10
3c	6	21.5	25.8	0.45	19.8	1.7	0	10
3d	6	21.9	25.4	0.46	20.9	1.0	0	10
3e	6	21.5	25.7	0.45	19.8	1.7	0	10
4a	10	40.4	66.0	0.40	39.8	0.7	0.0	10
4b	10	30.0	68.9	0.32	26.3	3.5	0.2	10
4c	10	30.9	74.4	0.31	29.0	1.9	0.0	10
4d	10	31.6	73.7	0.31	27.4	3.5	0.8	10
4e	10	31.6	73.8	0.30	29.8	1.8	0.0	10
5a	6	30.0	57.4	0.35	29.3	0.7	0.0	15
5b	6	21.4	64.9	0.24	17.7	3.4	0.2	15
5c	6	25.9	60.3	0.30	22.5	3.4	0.04	15
5d	6	24.8	61.4	0.28	21.0	3.7	0.04	15
5e	6	25.5	60.7	0.29	22.4	3.2	0.0	15

Table 2: Comparison of current method (label a) with four previous methods called coordinated-distracted (b), coordinated-dedicated (c), local-distracted (d) and local-dedicated (e). The abbreviations are same as in table 1.

V CONCLUSIONS

A method for motion strategy computation of a sensor that maximizes the number of target detections for the next T-time steps is presented. The method makes use of a-priori known statistics of target arrivals along with detections reported for the current sample to estimate the number of detections on a lattice of cells. To reduce overlaps a coordination mechanism is specified that performs a constrained search, where the

constraints are in the form of priorities assigned to sensors. The absence of an exhaustive search in the joint space due to the constraints renders the algorithm sub-optimal from a multi sensor perspective although optimal with respect to a single sensor in presence of those constraints. The tabulations presented in the results section vindicate the performance of the current approach in comparison to previous approaches for target detection and pursuit. The performance enhancement due to the coordination phase that reduces overlaps in FOV between sensors is also tabulated in section IV.

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