Graph Theory

Assignment 6
Kishore Kothapalli

Due: 15-APR-2009

Problem 1. Call a graph as k–degenerate if every subgraph of G has a vertex of degree at most k. Show that every planar graph is 5–degenerate. Show that every k-degenerate graph can be colored with no more $k+1$ colors. **(2 Points)**

Problem 2. Prove that a plane graph G is bipartite if and only if every face has an even length. (Hint: Use inductio on the number of faces.) **(3 Points)**

Problem 3. Read about the Petersen graph. Show that the Petersen graph is not planar in two ways: (a) Using the Kuratowski theorem, and (b) by using Euler's formula for graphs of girth 5.
(3 Points)

Problem 4. Find the smallest n such that the complement of a simple plane graph with at least n vertices is non-planar. **(3 Points)**

Problem 5. Prove or disprove: There is no simple bipartite planar graph with minimum degree at least 4.
(2 Points)