# Graph Theory

Assignment 5
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Due: 8-APR-2009

**Problem 0.** Given a graph $G$ with vertex set $V = \{v_1, v_2, \cdots, v_n\}$, let $G'$ be the graph generated from $G$ using the Mycielski's construction. Let $H$ be a subgraph of $G$. Let $G''$ be the graph obtained from $G'$ by adding edges $\{u_iu_j : v_iv_j \in E(H)\}$. Prove that $\chi(G'') = \chi(G) + 1$ and $\omega(G'') = \max\{\omega(G), \omega(H) + 1\}$ **(4 Points)**

**Problem 1.** Show the following properties of flow in a network $G$.

- For all $A \subseteq V$, $f(A, A) = 0$.
- For all $A, B \subseteq V$, $f(A, B) = -f(B, A)$.
- For all $A, B, C \subseteq V$ with $A \cap B = \Phi$, it holds that $f(A \cup B, C) = f(A, C) + f(B, C)$ and $f(C, A \cup B) = f(C, A) + f(C, B)$.

**(3 Points)**

**Problem 2.** Does the construction for maximum matching in bipartite graphs using network flows work also for the case of weighted bipartite matching. In the weighted bipartite matching, we seek to find a matching of maximum weight. The weight of a macthing is the sum of the weights of the edges in the matching. Justify your answer. **(2 Points)**

**Problem 3.** What is an upper bound on the length of any augmenting path in the flow network $G'$ constructed for a bipartite graph $G$ so as to find the maximum matching in $G$. **(2 Points)**

**Problem 4.** Let $G_f$ be a residual network and $P$ be an augmenting path with residual capacity $c_f(P)$. Show that the function $f_P$, below defines a flow in $G_f$.

$$f_P = \begin{cases} c_f(P) & \text{if } (u, v) \text{ is on } P \\ -c_f(P) & \text{if } (v, u) \text{ is on } P \\ 0 & \text{otherwise} \end{cases}$$

**(3 Points)**