

Graph Theory

Assignment 4
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Due: 2-APR-2009

Problem 18. Prove that the deterministic algorithm for constructing a bipartite subgraph H of a given graph G will ensure that $|E(H)| \geq |E(G)|/2$. (Hint: Use induction on n). **(3 Points)**

Problem 19. Think of the proof of Tutte's theorem regarding 1-factors in general graphs. How can we guarantee the presence of vertices x, y, z in the proof as discussed. **(3 Points)**

Problem 19. Prove or disprove: For every graph G , $\chi(G) \leq n(G) - \alpha(G) + 1$. **(2 Points)**

Problem 20. Prove that every triangle free graph with n vertices is colorable using $2\sqrt{n}$ colors **(5 Points)**

Problem 21. Let $V = \{0, 1\}^d$ be the vertex set of a graph and the edge set is the set of vertex pairs that differ exactly at one bit position. Find $\chi(G)$ and a colouring using $\chi(G)$ colours. **(3 Points)**

Problem 22. Prove that every graph has a vertex ordering relative to which greedy coloring uses $\chi(G)$ colors. **(3 Points)**

Problem 23. Give a constructive proof of the following statement.

Every graph G can be coloured in no more than \sqrt{m} colours.

(2 Points)