Graph Theory

Assignment 1
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Due: 22-JAN-2009

Problem 1. Suppose $d = (d_1, d_2, \cdots, d_{2k})$ is defined by $d_{2i} = d_{2i-1} = i$ for $1 \leq i \leq k$. Prove that d is graphic. **(3 Points)**

Problem 2. Prove of disprove:
In the BFS of a directed graph, there are no cross edges. In the BFS of an undirected graph there are no forward edges. **(2 Points)**

Problem 3. Prove or disprove the following claim:

> In a DFS of a directed graph G, if there is a path from u to v and if $f(u) > f(v)$ then v is a descendant of u in the resulting DFS forest.

(2 Points)

Problem 4. In a DFS of a directed graph G,if $S = (d[u] - d[v]) * (f[v] - f[u])$, comment on the nature of edge uv with regard to the sign of value of S,with S being nonzero.
(2 Points)

Problem 5. There are two types of professional wrestlers: "good guys" and "bad guys".Between any pair of professional wrestlers,there may or may not be a rivalry.Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries.Given an $O(n + r)$ algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy.If it is possible to perform such a designation,your algorithm should produce it.
(3 Points)

Problem 6. Take an undirected graph of about 15 vertices and 20 edges and identify the bi-connected components of your graph using the algorithm presented in the lecture. **(2 Points)**

Problem 7. Read the definition of an edge cut of the form $[S, \overline{S}]$ from the lecture notes posted. Then solve the following problems.

- Compute $|[S, \overline{S}]|$ for $K_{m,n}$ when S has a vertices from one partite set and b from the other.
- Compute the maximum no of edges possible in an R partite graph.

(2 Points)

Problem 8. Give a Modified DFS algorithm which computes the low-points of all the vertices.
(4 Points)