**Problem 1.** Consider a directed graph having at least 15 vertices and 25 edges. Perform a DFS on the graph you consider, identify the start and finish times of all vertices, and classify edges according to the DFS. **(10 Points)**

**Problem 2.** For the graph you consider in Problem 1, perform a BFS and classify edges according to the BFS. **(10 Points)**

**Problem 3.** Give an example of a graph with negative edge weight(s) where Dijkstra's algorithm fails to work. Where in the proof of Dijkstra's algorithm do we require that edge weights are not negative? **(10 Points)**

**Problem 4.** Work out an example of Dijsktra's algorithm on a directed graph of about 8 vertices 15 edges. Show all your work. **(10 Points)**

**Problem 5.** Read the definition of the component graph of a directed graph from the book (CLRS). Given a directed graph $G$, give an $O(n+m)$ time algorithm to compute the component graph of $G$. Your component graph should contain only one edge between any two vertices in the component graph.
**(10 Points)**

**Problem 6.** Work out an example of the Bellman-Ford algorithm on a directed graph of about 5 vertices 8 edges. Show all your work. **(10 Points)**

**Problem 7.** Let $G$ be a directed graph with $(u, v) \in E(G)$. Prove of disprove:
(a) In any DFS of $G$, it is always true that $f(u) > f(v)$?
(b) In any BFS of $G$, it is always true that $(u, v)$ is a tree edge?
**(10 Points)**