# CS 3100–Algorithms

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**Problem 1** *Let $L$ be a given line that represent a long hall way in an art gallery. We are also given a set $X = \{x_0, \ldots, x_{n-1}\}$ of real numbers that specify the position of paintings in the hallway. A single guard can protect all paintings within distance 1 of his/her position (on both the sides). Design an algorithm for finding a placement of guards that uses minimum number of guards to guard all the paintings with positions in $X$.*

*Also prove the correctness of your algorithm. (Marks 10)*

**Problem 2** *Consider a binary tree $T$ such that for any node $u \in T$, we have an weight $w(u)$ assigned to it. An independent set of $T$ is a subset $S$ of nodes of $T$ such that no nodes in $S$ is a child or parent of any other node in $S$. Design a dynamic programming algorithm to find the maximum weight independent set of nodes in $T$ where the weight of a set of nodes is simply the sum of the weights of the nodes in the set.*

*Prove the correctness of your algorithm. (Marks 10).*

**Problem 3** *Assume that we have a machine $m$ and we are given a set $T$ of tasks specified by the start time and finish time. Unfortunately we have an additional constraint that no two jobs can be executed simultaneously. Design a greedy algorithm to maximize the number of tasks this single machine can perform.*

*Give the correctness proof for your algorithm. (Marks 10).*

**Problem 4** *Work out an example with about 6 jobs to select a subset of these jobs with maximum profit for scheduling . Use the dynamic programing based solution and show all your work. (Marks 5)*

**Problem 5** *Suppose we are given a collection* $A = \{a_1, \ldots, a_n\}$ *of* $n$ *positive integers that add up to* $N$*. Design an efficient algorithm to determine if there exits a subset* $B \subset A$*, such that* $\sum_{a_j \in B} a_j = \sum_{a_i \in A-B} a_i$*. (Marks 10)*

**Problem 6** *Describe an efficient algorithm for making change for a specified value with minimum number of coins assuming there are four denominations of coins with values* $25, 10, 5$ *and* $1$ *respectively. Argue why your solution is correct. (Marks 5)*