

# Homework 5

## Complexity and Advanced Algorithms

Due November 25, 2011.

**Problem 1.** Revisit the MIS algorithm discussed in the lecture. An alternative proof technique is to try to show that the number of nodes reduce by a constant fraction in each iteration. Show that this proof technique is difficult to work with by showing an example run on the algorithm on some graph where the number of nodes does not decrease by a constant fraction. **(3 Points)**

**Problem 2.** The MIS algorithm produces an MIS, but not a maximum independent set. How far away can the size of the MIS produced by the algorithm be from that of a maximum independent set? Are there example graphs where the MIS produced has a size that is only  $\Theta(1/n)$  of the size of a maximum independent set where  $n$  is the number of nodes in the graph? Are there examples where the ratio of the size of MIS to that of a maximum independent set is  $\Theta(1)$ ? **(2 Points)**

**Problem 3.** Read the proof of the claim corresponding to the lower bound of computing the OR of  $n$  bits on a CREW PRAM. Prove Lemma 10.2.2. **(3 Points)**

**Problem 4.** What is a critical input for the Boolean AND function. Similarly, what is a critical input for the Boolean XOR function. These functions have the standard meaning. **(2 Points)**