Homework 5
Complexity and Advanced Algorithms

Due November 4, 2011.

Problem 1. In the expression tree evaluation using the rake technique, consider raking a leaf node u with sibling v and parent w. Show how to update the labels on node v when the operator at node w is (a) subtraction, (b) division, and (c) exponentiation. In case of (b) and (c) above, you may have to think of additional labels. **(4 Points)**

Problem 2. The aim of this problem is to show how to solve the LCA problem on complete binary trees. Let T be a complete binary tree with $n = 2^k - 1$ vertices. Let each node be given a number according to the inorder traversal. For any node u, let $r(u)$ be defined as the index of the rightmost bit position with value 1. (Bit positions start with the LSB as index 0, and increase towards the left). Show the following:

1. All the descendants of u have the same $r(u) - k$ leftmost bits.
2. No descendant of u has more than k consecutive righmost 0 bits, and
3. Given two nodes u and v in T, statements (1) and (2) above can be used to find the LCA of u and v.

(4 Points)

Problem 3. Use the Euler tour of a rooted binary tree to compute the inorder number of nodes in tree. The inorder number of a node is the index of the node in the inorder traversal of the tree. Your algorithm should use only $O(n)$ operations. **(3 Points)**

Problem 4. Let T be a rooted tree. How many times would a node appear in an Euler path of T. **(2 Points)**

Problem 5. Find classes of graphs which have: (a) an indpendent set that is a constant fraction of the number of nodes,
(b) an independent set that has only $O(1)$ size irrespective of the number of nodes in the graph. **(2 Points)**