Homework 5
Complexity and Advanced Algorithms

Due October 4, 2011.

Problem 1. Obtain the space complexity of the perfect hashing scheme, not using asymptotics. You should include the space for storing hash functions etc. Also, obtain the worst case time complexity of hashing n elements assuming that applying the hash function is an $O(1)$ operation. (Hint: For the space complexity part, you should count the number of hash functions possible). **(4 Points)**

Problem 2. Let Φ be a Boolean formula in conjunctive normal form (CNF) of n clauses with each clause having $k \geq 2$ variables. Suppose that a Boolean assignment of truth values to the Boolean variables to chosen so that each variable is set to T or F uniformly at random and independent of the truth value of the other random variables. Compute the expected number of clauses satisifed by such a truth assignment. (Hint: Use random variables)

Problem 3. Suppose the two sequences we wish to merge A, B do not have distinct elements. Show how to create distinct elements so that the designed algorithms can be used. **(2 Points)**

Problem 4. Prove the following claims wrt a doubly logarithmic tree of n nodes.

- It has a depth of $O(\log \log n)$.
- The number of nodes at level i is $2^{2^k - 2^{k-i}}$ for $0 \leq i < k$.

(4 Points)

Problem 5. Suppose we have two algorithms A and B to solve a problem P of size n. Algorithm A takes $O(\log n)$ time on the PRAM using $O(n \log n)$ operations. Algorithm B reduces the size of P by a constant factor in $O(\log n / \log \log n)$ time using $O(n)$ operations. Devise an $O(\log n)$ time algorithm for P that uses only $O(n)$ operations. **(4 Points)**