Homework 3
Complexity and Advanced Algorithms

Due September 6, 2011.

Problem 1. Recall the proof of the theorem on gaps in space. Essentially, we showed that given strictly more space, one can recognize more languages. In the proof, where do we require $f(n)$ space? Would $o(f(n))$ space suffice? Justify your answers. Further, why do we need $f(n)$ to be space constructible? **(4 Points)**

Problem 2. Give an example of a sparse regular language and a non-sparse regular language. Similarly, give an example of a sparse context free language, and a non-sparse context free language. Justify your examples. As much as possible, avoid unnatural examples. **(4 Points)**

Problem 3. Let a fair coin be tossed n times for a positive integer n. Let X denote the number of heads. Compute $E(X^2)$ and $E(X^4)$. **(4 Points)**

Problem 4. Recall the definition of conditional probability. Argue why the definition is a valid definition of probability. **(4 Points)**