

Homework 2

Complexity and Advanced Algorithms

Due August 19, 2011.

Problem 1. Show that the problem of deciding whether a graph is bipartite is in NLogSpace. Is it in LogSpace?

Problem 2.

Problem 2. The problem of graph isomorphism is defined as follows. Let G_1 and G_2 be two graphs. G_1 is said to be isomorphic to G_2 if there exists a function $f : V(G_1) \rightarrow V(G_2)$ such that $vw \in E(G_1)$ iff $f(v)f(w) \in E(G_2)$ for all $v, w \in V(G_1)$. Justify whether the graph isomorphism problem can be decided by a TM using (a) nondeterministic polynomial time, (b) deterministic polynomial space, and (c) nondeterministic logarithmic space. **(5 Points)**

Problem 3. Express the following statements about natural numbers as quantified formulae.

- Every two natural numbers have at least one common divisor.
- For no integer k more than 2, it holds that the k th power of two distinct integers x and y equals the k th power of another integer z . (Note: This theorem is called as the Fermat's theorem and the case of $k = 2$ gives rise to Pythagoras theorem. It is shown only in the last decade or so that for no other value of k such property exists.)
- A small change from the above. Now, 4-tuples of integers and an integer k such that the sum of a pair of integers in the above 4-tuple raised to their k th power equals the sum of the remaining pair of integers raised to their k th power. (Note: You may have heard of one of the last discoveries of the great mathematician Ramanujam where he says that 1729 can be expressed as the sum of cubes of two distinct pairs of numbers. What are the pairs?)

(3 Points)