

Homework 1

Complexity and Advanced Algorithms

Due August 9, 2011.

Problem 1. Design a TM for computing the following functions.

- $f(n) = \lceil \log n \rceil$.
- $f(n) = \lfloor \sqrt{n} \rfloor$.

(2 × 2 = 4 Points)

Problem 2. Consider a pushdown automata (PDA) that is equipped with two stacks with the restriction that the size of the stacks is bounded by a polynomial in the length of the input. What is the set of languages that can be accepted by such a modified PDA. Justify your answer. (4 Points)

Problem 3. Consider the satisfiability problem in which each clause has only 2 literals. Call the corresponding language as 2SAT. Is 2SAT in NP? Is it NP-complete? Justify your answers. (4 Points)

Problem 4. Solve the recurrence relation below for all possible values of a_1 and a_2 being positive real numbers.

$$T(n) = T(a_1n) + T(a_2n) + O(n)$$

Show all your work and justify your answers. (4 Points)

Problem 5. Consider a set of n processes with running times r_1, r_2, \dots, r_n units. To execute these, there is one machine M with an available running time of R units. Design an algorithm to select a set of processes from the above so that their total running time is at most and as closest to R . Argue why your algorithm is correct. (4 Points)