Graph Theory

Assignment 5
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Due: 8-APR-2009

Problem 0. Given a graph G with vertex set $V = \{v_1, v_2, \cdots, v_n\}$, let G' be the graph generated from G using the Mycielski's construction. Let H be a subgraph of G. Let G'' be the graph obtained from G' by adding edges $\{u_iu_j : v_iv_j \in E(H)\}$. Prove that $\chi(G'') = \chi(G) + 1$ and $\omega(G'') = \max\{\omega(G), \omega(H) + 1\}$ **(4 Points)**

Problem 1. Show the following properties of flow in a network G.

- For all $A \subseteq V$, $f(A, A) = 0$.
- For all $A, B \subseteq V$, $f(A, B) = -f(B, A)$.
- For all $A, B, C \subseteq V$ with $A \cap B = \Phi$, it holds that $f(A \cup B, C) = f(A, C) + f(B, C)$ and $f(C, A \cup B) = f(C, A) + f(C, B)$.

(3 Points)

Problem 2. Does the construction for maximum matching in bipartite graphs using network flows work also for the case of weighted bipartite matching. In the weighted bipartite matching, we seek to find a matching of maximum weight. The weight of a macthing is the sum of the weights of the edges in the matching. Justify your answer. **(2 Points)**

Problem 3. What is an upper bound on the length of any augmenting path in the flow network G' constructed for a bipartite graph G so as to find the maximum matching in G. **(2 Points)**

Problem 4. Let G_f be a residual network and P be an augmenting path with residual capacity $c_f(P)$. Show that the function f_P, below defines a flow in G_f.

$$f_P = \begin{cases} c_f(P) & \text{if } (u, v) \text{ is on } P \\ -c_f(P) & \text{if } (v, u) \text{ is on } P \\ 0 & \text{otherwise} \end{cases}$$

(3 Points)