Sources of Optimization

- In order to optimize our IR, we need to understand why it can be improved in the first place.
- **Reason one:** IR generation introduces redundancy.
  - A naïve translation of high-level language features into IR often introduces subcomputations.
  - Those subcomputations can often be sped up, shared, or eliminated.
- **Reason two:** Programmers are lazy.
  - Code executed inside of a loop can often be factored out of the loop.
  - Language features with side effects often used for purposes other than those side effects.

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
bool b3;

b1 = x + x < y
b2 = x + x == y
b3 = x + x > y
```

Optimizations from IR Generation

```
int x;
int y;
bool b1;
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bool b3;

b1 = x + x < y
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b3 = x + x > y
```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

_t2 = x + x;
_t3 = y;
b2 = _t2 == _t3;

_t4 = x + x;
_t5 = y;
b3 = _t5 < _t4;
```

Optimizations from IR Generation

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_t0 = x + x;
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b3 = _t0 < _t1;
```

```
while (x < y + z) {
    x = x - y;
}
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
```

```
_L0:
    _t0 = y + z;
    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
    x = x - y;
    Goto _L0;
_L1:
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
```

```
_L0:
    _t0 = y + z;
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Optimizations from Lazy Coders

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while (x < y + z) {
    x = x - y;
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    _t0 = y + z;
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    _t1 = x < _t0;
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_L1:
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Optimizations from Lazy Coders

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while (x < y + z) {
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    _t0 = y + z;
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    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
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    Goto _L0;
_L1:
```

A Note on Terminology

- The term “optimization” implies looking for an “optimal” piece of code for a program.
- This is, in general, undecidable.
  - e.g. create a program that can be simplified iff some other program halts.
- Our goal will be IR *improvement* rather than IR *optimization*.

The Challenge of Optimization

- A good optimizer
  - Should never change the observable behavior of a program.
  - Should produce IR that is as efficient as possible.
  - Should not take too long to process inputs.
- Unfortunately:
  - Even good optimizers sometimes introduce bugs into code.
  - Optimizers often miss “easy” optimizations due to limitations of their algorithms.
  - Almost all interesting optimizations are **NP**-hard or undecidable.

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?
- **Runtime** (make the program as fast as possible at the expense of time and power)
- **Memory usage** (generate the smallest possible executable at the expense of time and power)
- **Power consumption** (choose simple instructions at the expense of speed and memory usage)
- Plus a lot more (minimize function calls, reduce use of floating-point hardware, etc.)

Basic Blocks

- A **basic block** is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence.
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence.
- Informally, a sequence of instructions that always execute as a group.

Control-Flow Graphs

- A **control-flow graph** (CFG) is a graph of the basic blocks in a function.
  - The term CFG is overloaded – from here on out, we'll mean "control-flow graph" and not "context-free grammar."
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block.
- There is a dedicated node for the start and end of a function.

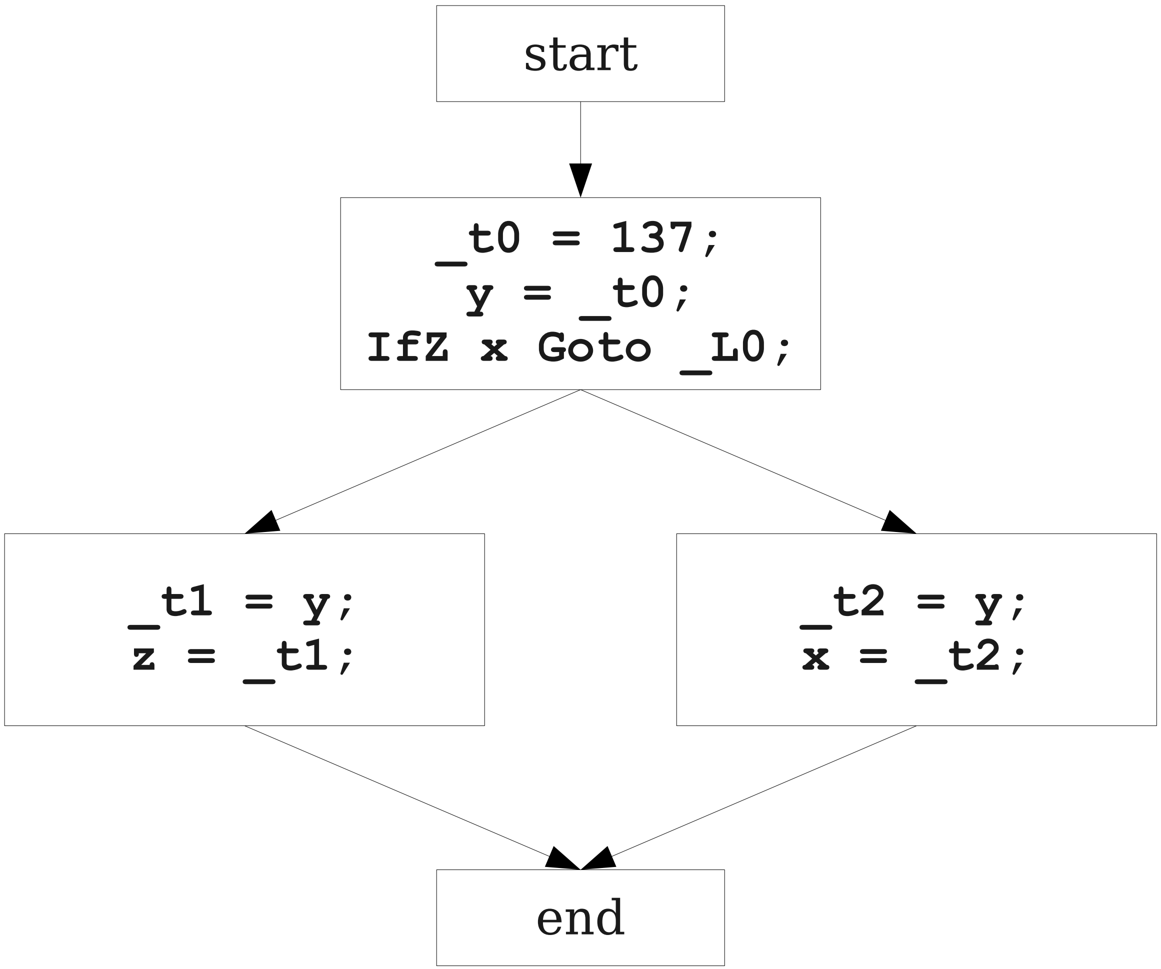
Types of Optimizations

- An optimization is **local** if it works on just a single basic block.
- An optimization is **global** if it works on an entire control-flow graph.
- An optimization is **interprocedural** if it works across the control-flow graphs of multiple functions.
  - We won't talk about this in this course.

Local Optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



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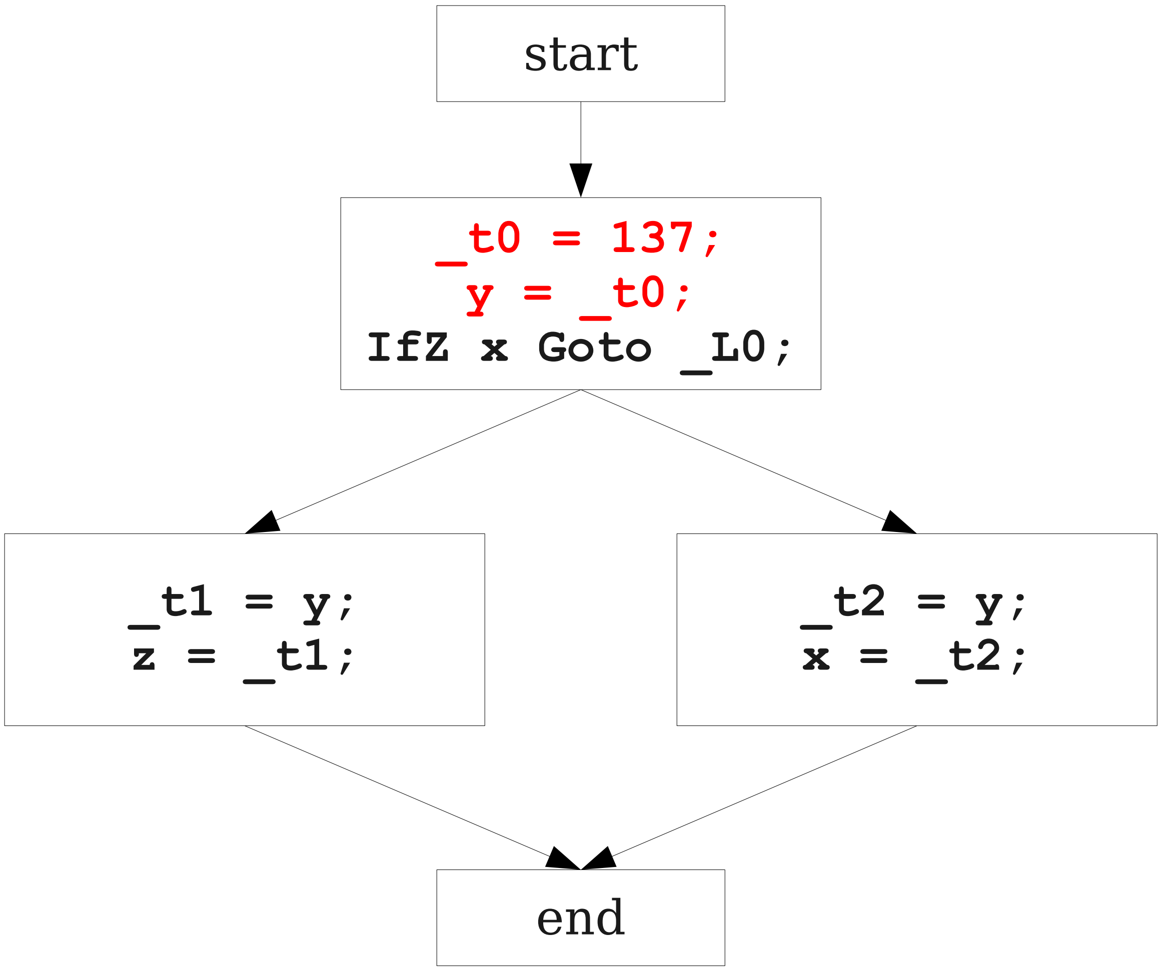

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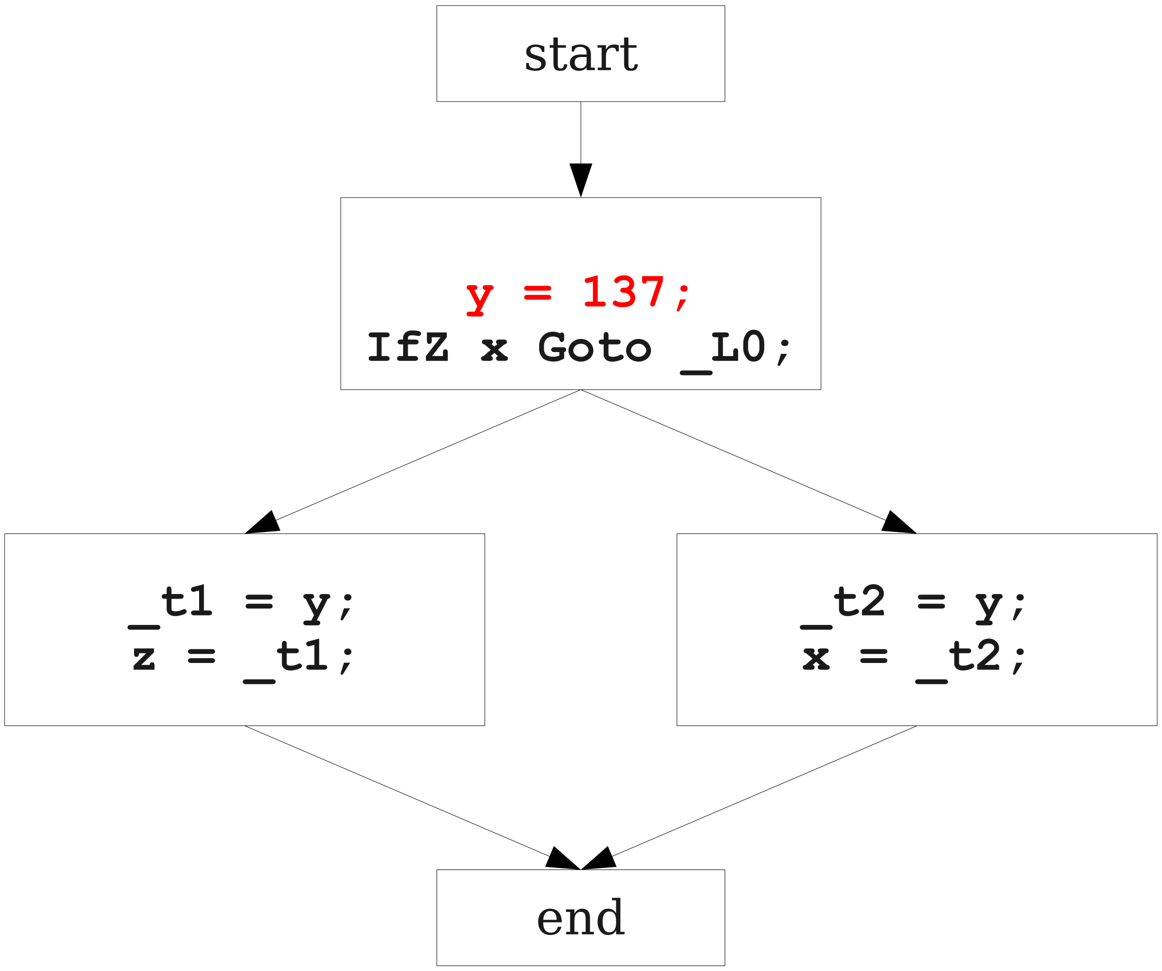

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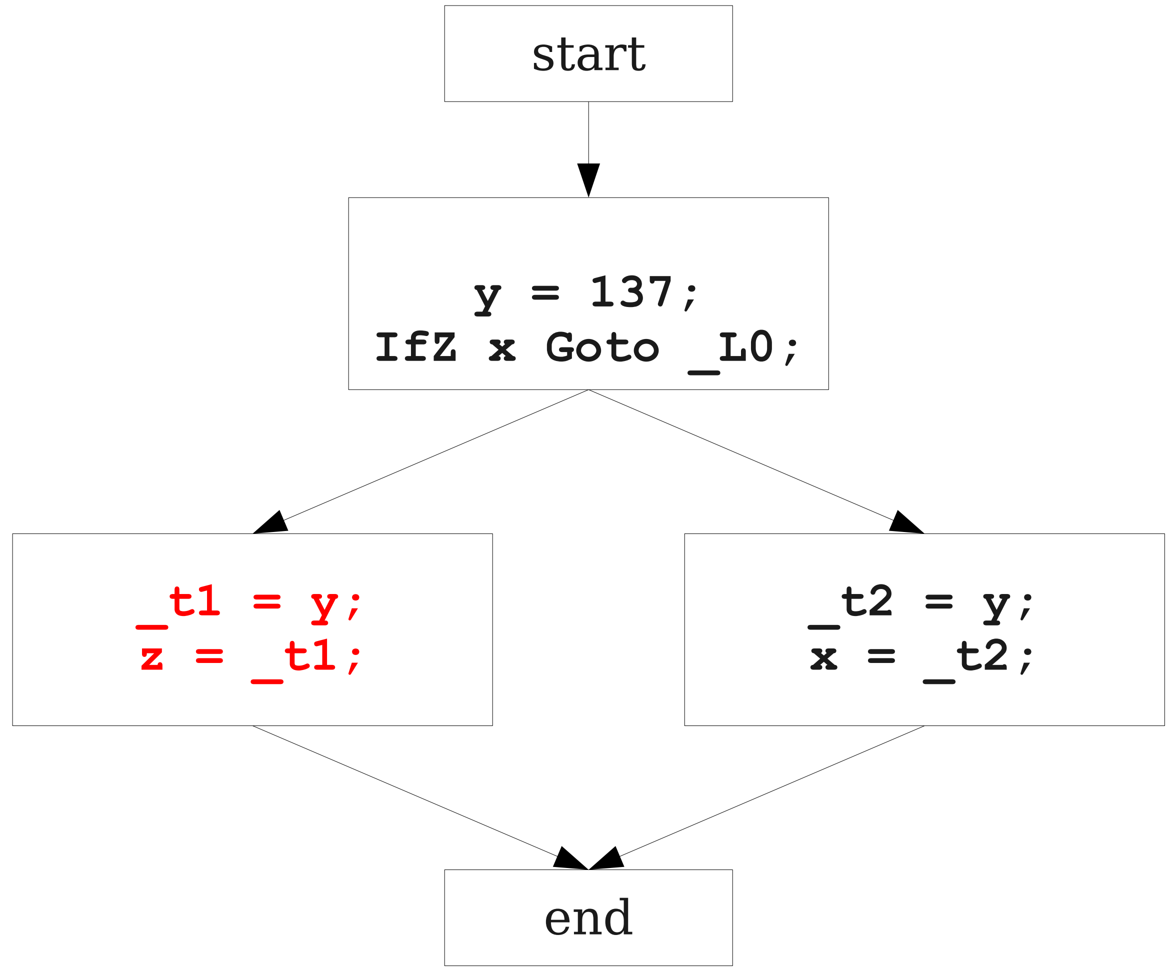

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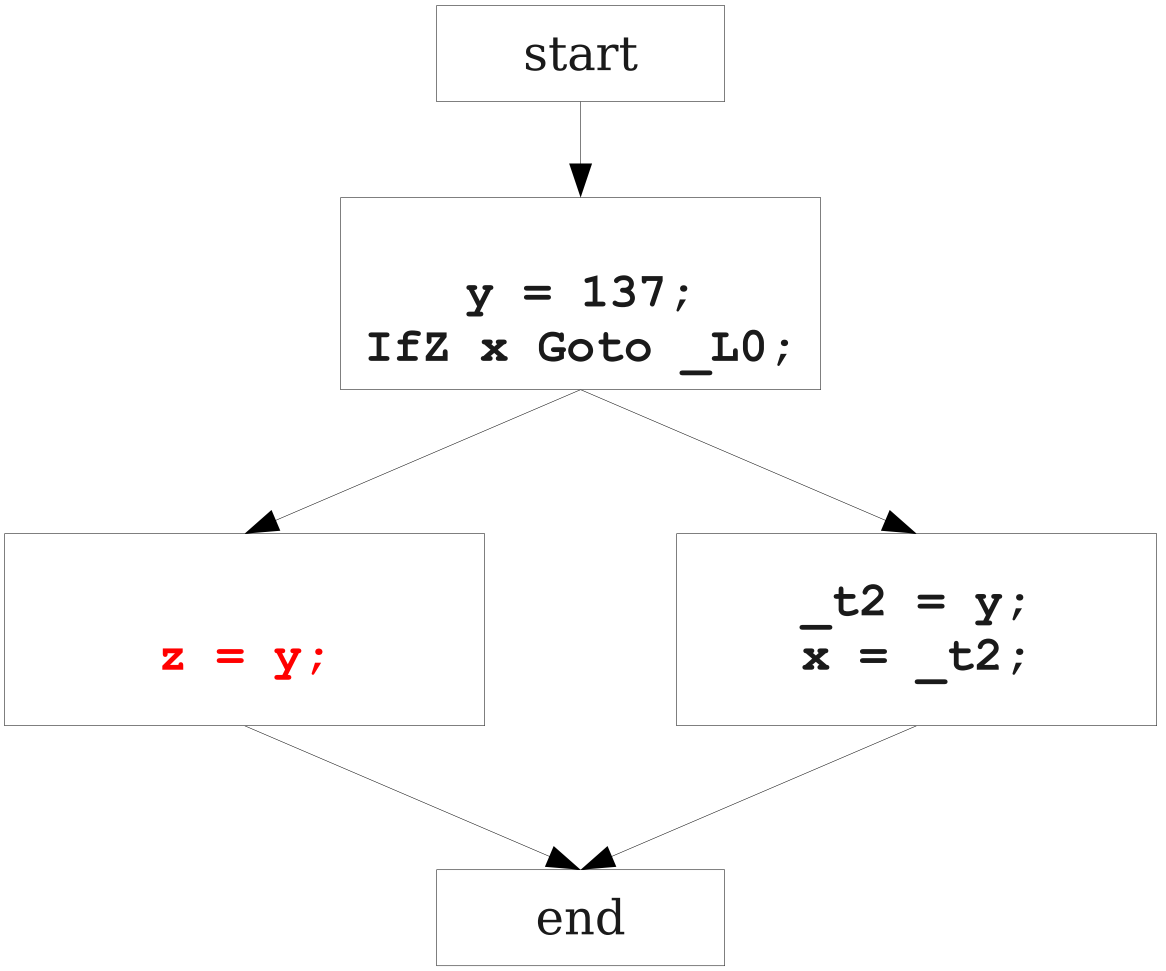

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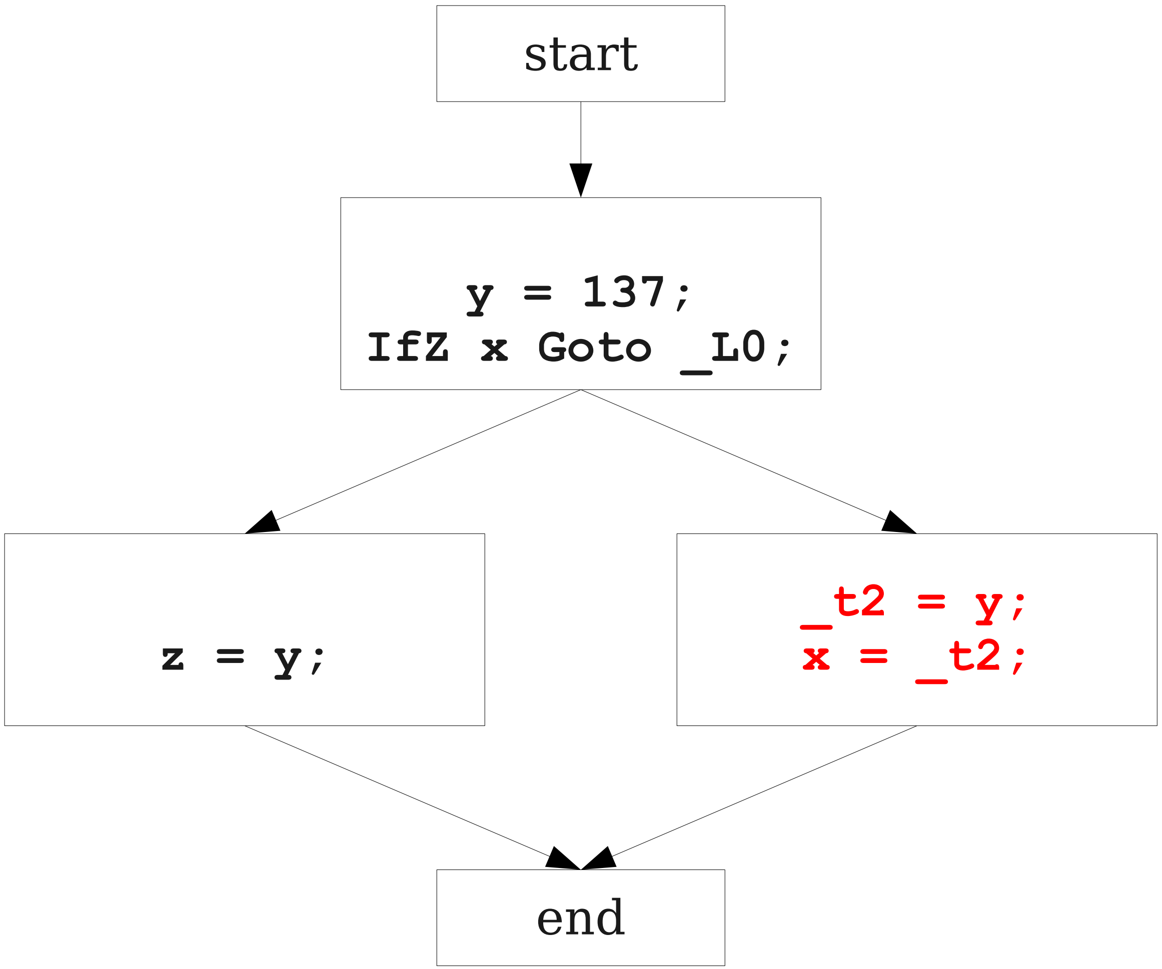

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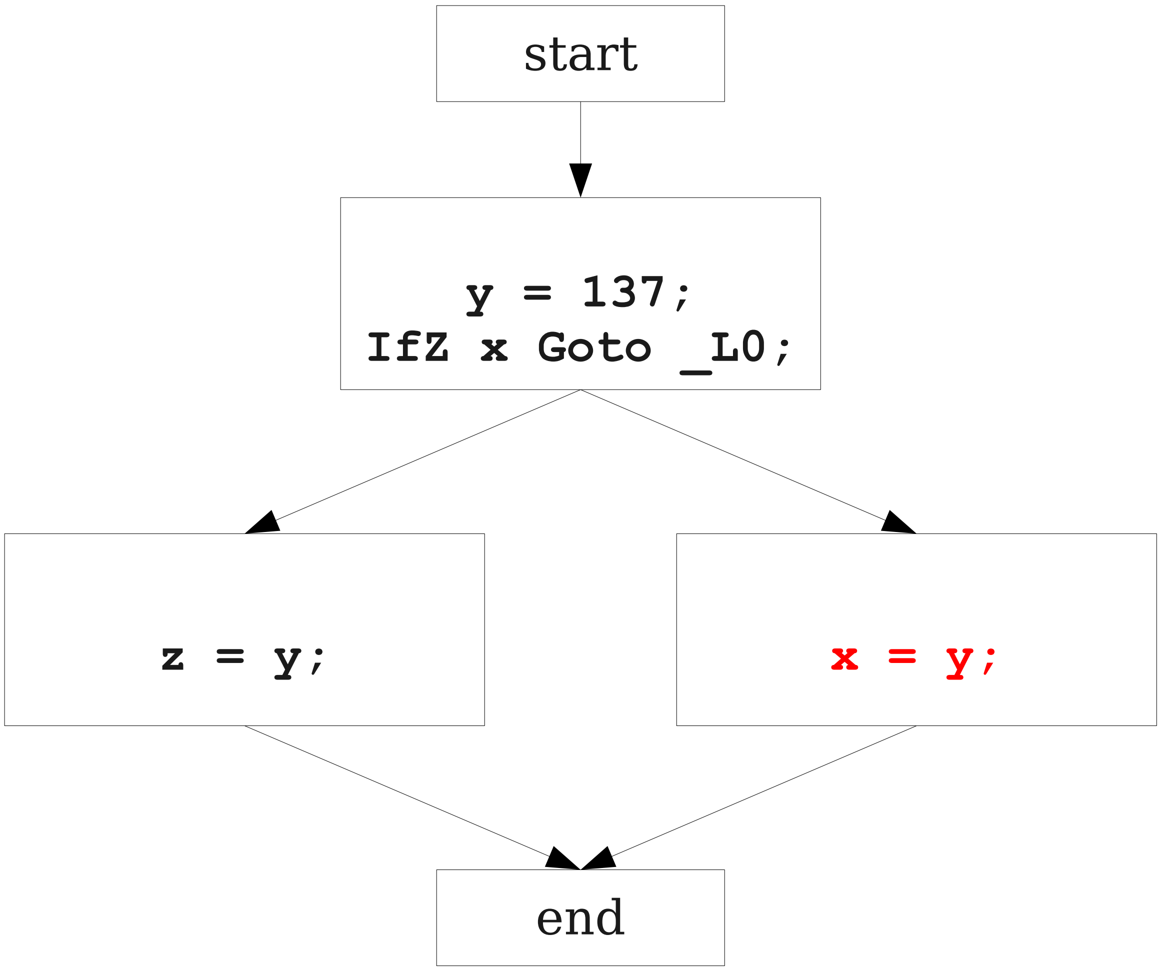

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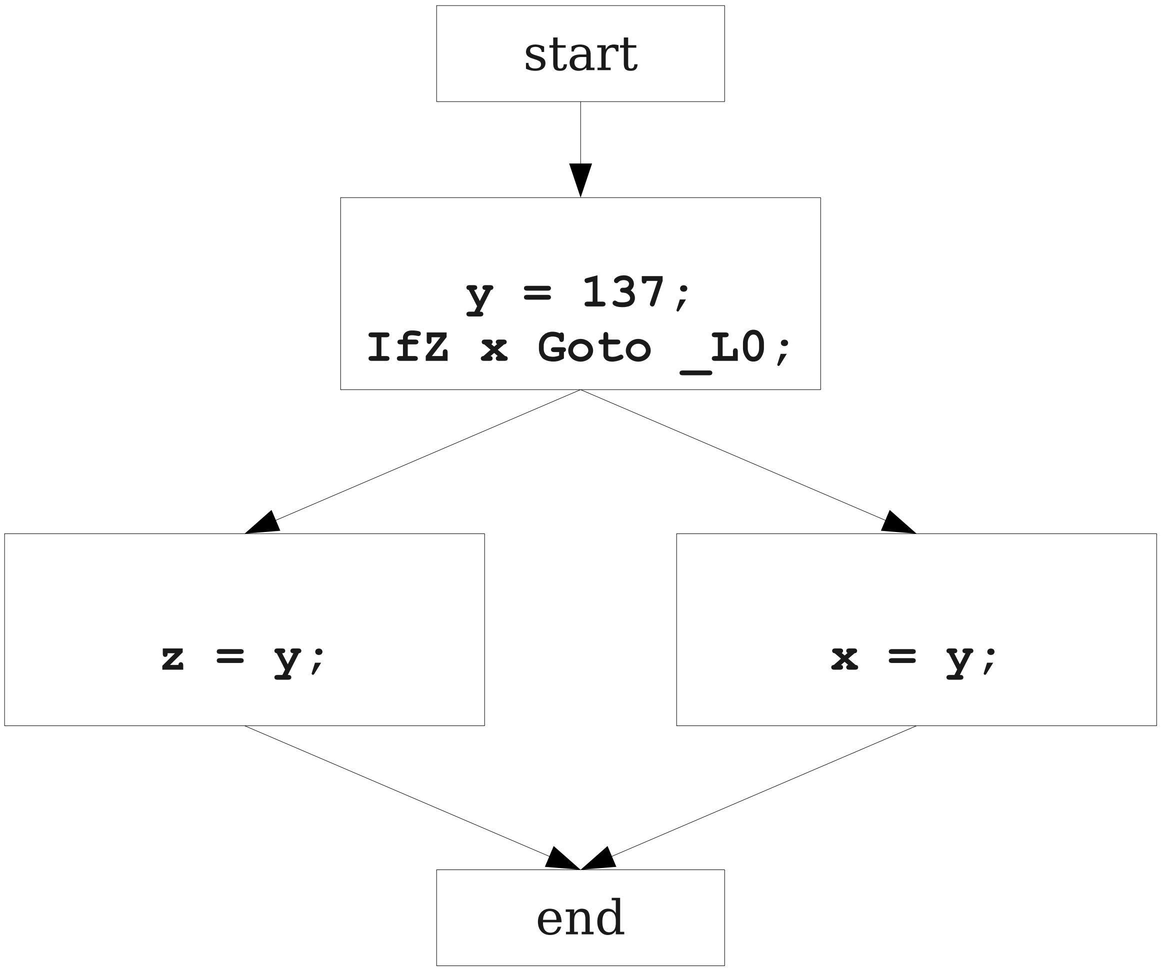

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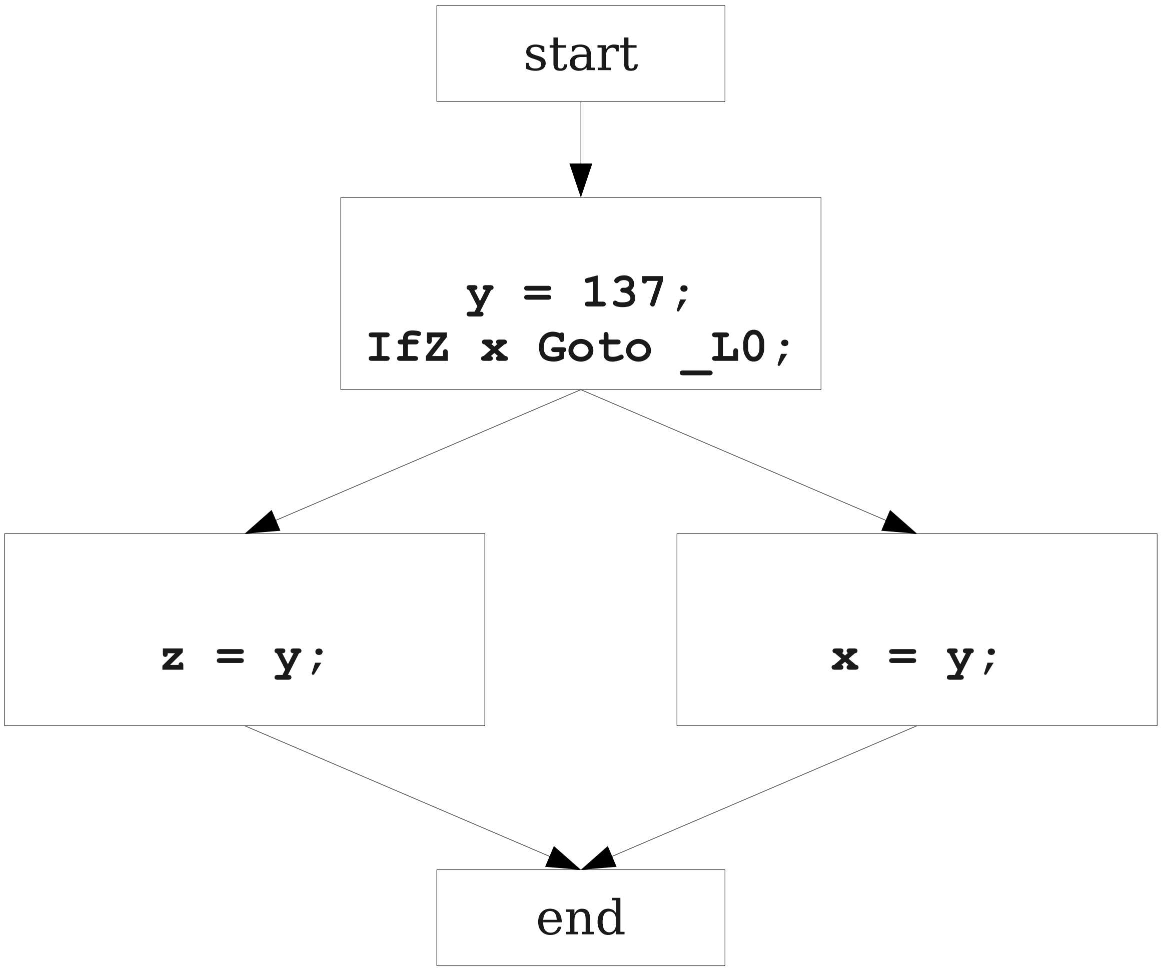

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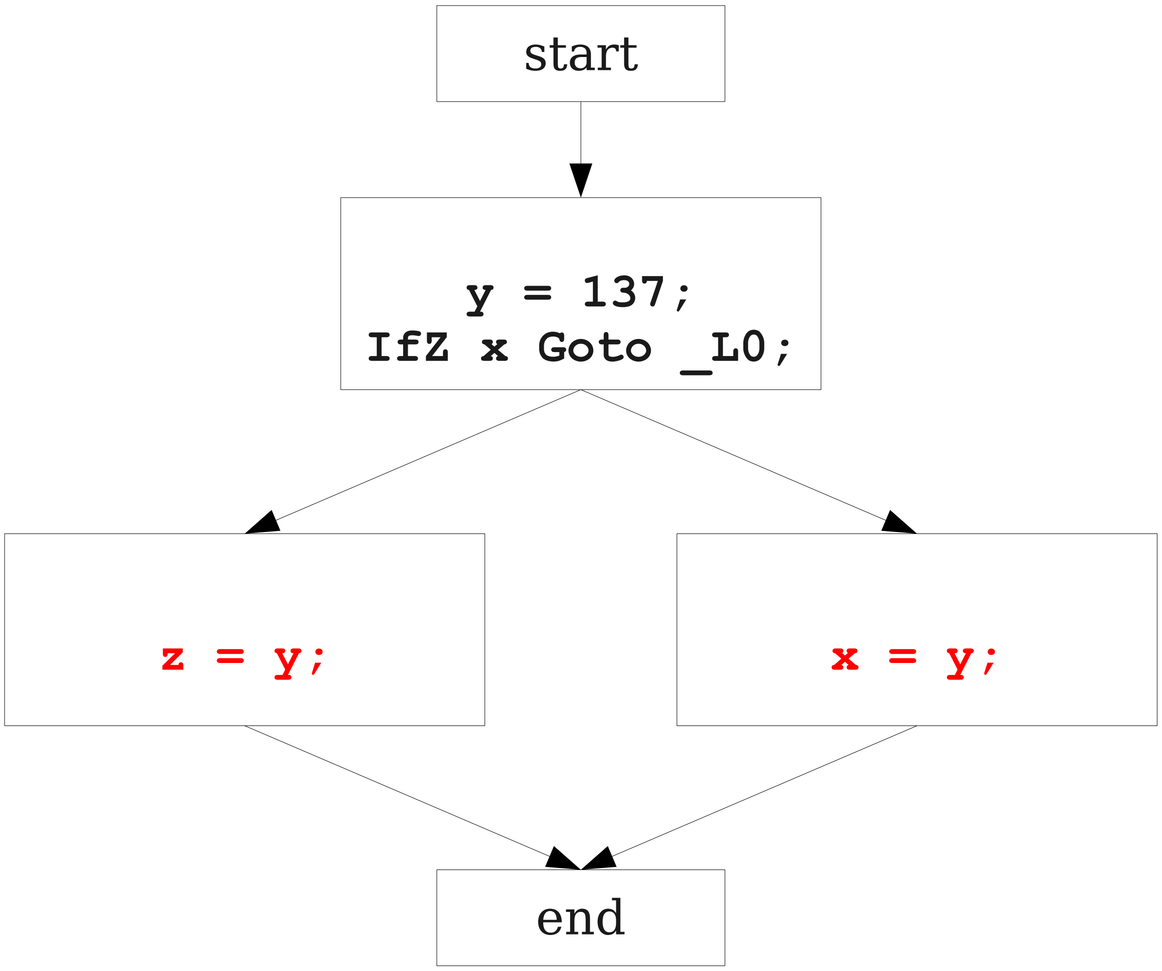

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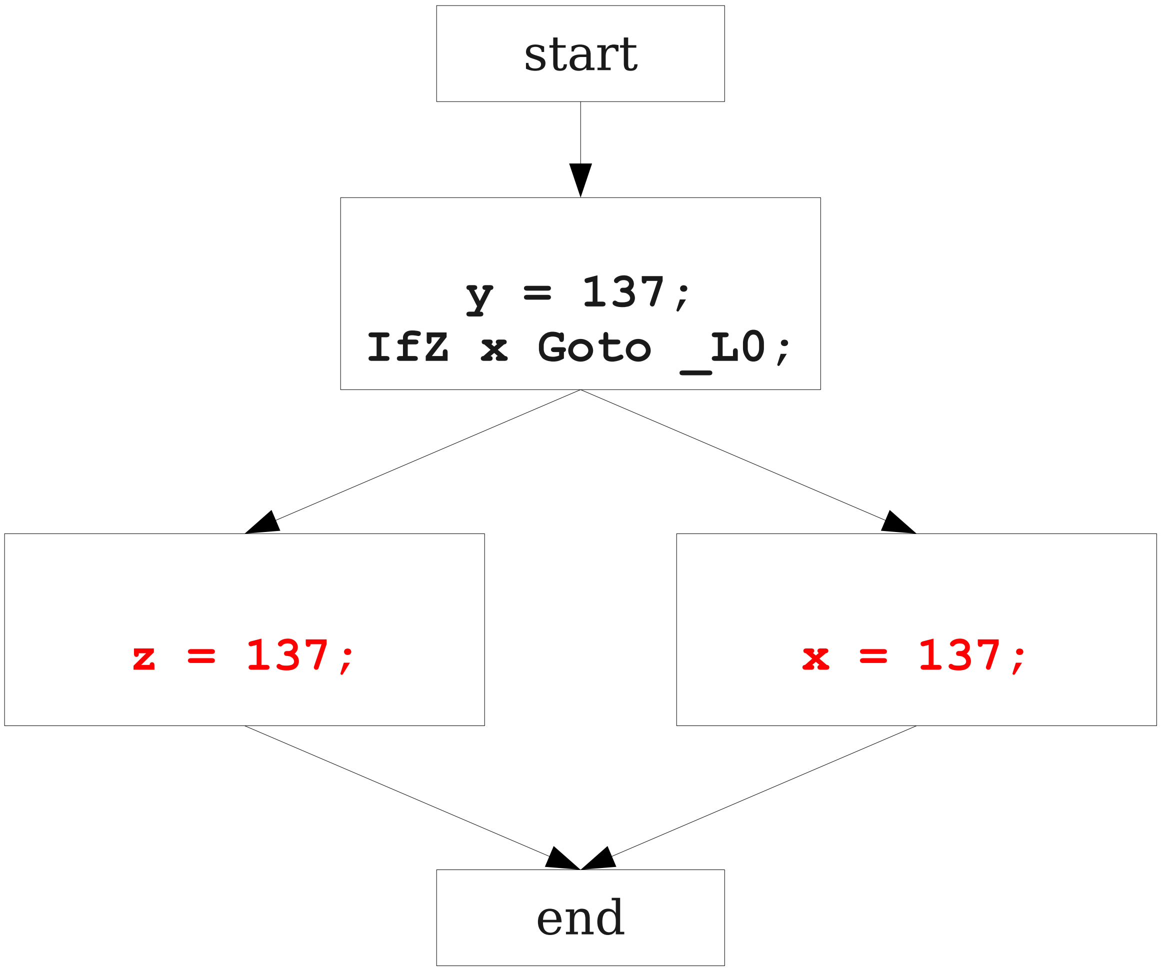

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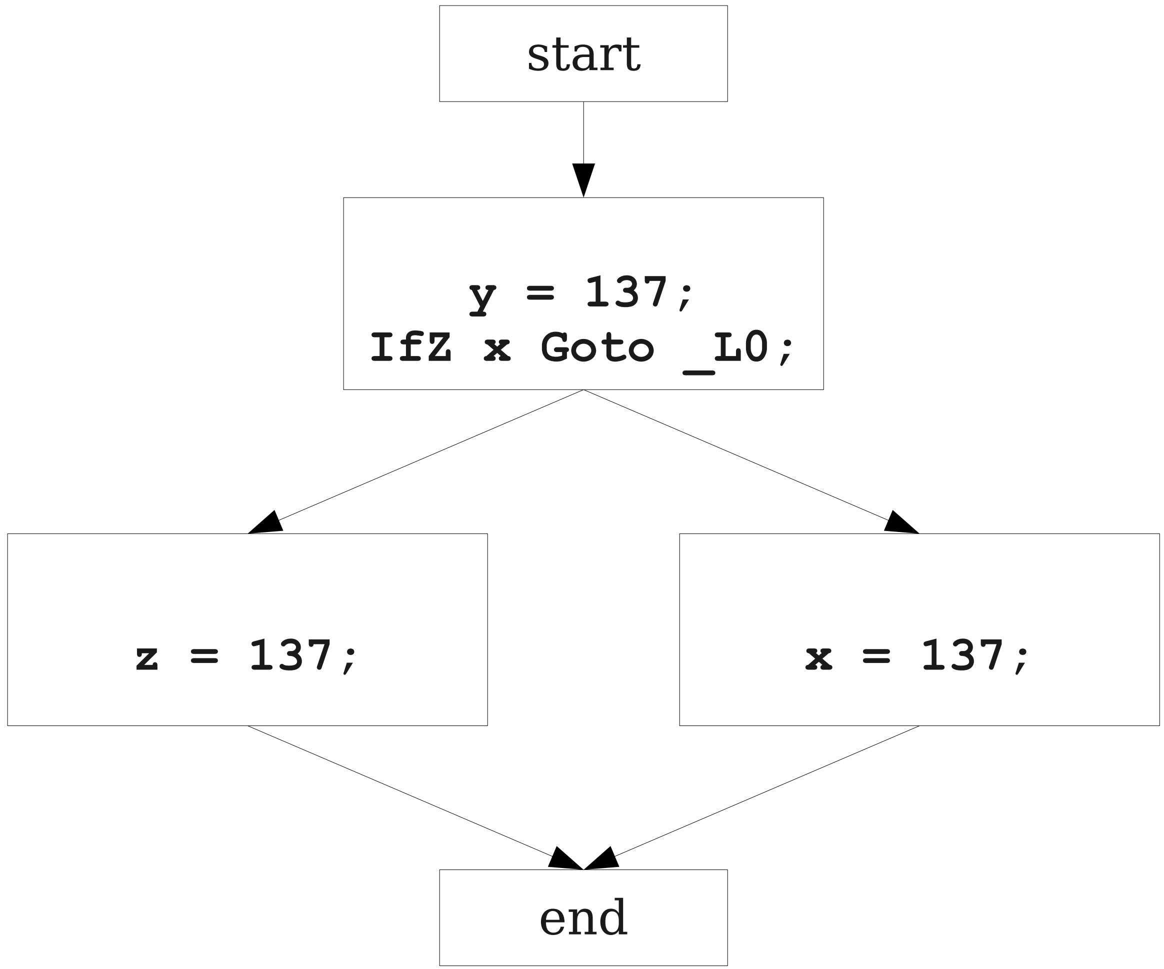

Local Optimizations

Common Subexpression Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

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Common Subexpression Elimination

- If we have two variable assignments

  $\mathbf{v_1 = a\ op\ b}$

  $\mathbf{\ldots}$

  $\mathbf{v_2 = a\ op\ b}$

  and the values of $\mathbf{v_1}$, **a**, and **b** have not changed between the assignments, rewrite the code as

  $\mathbf{v_1 = a\ op\ b}$

  $\mathbf{\ldots}$

  $\mathbf{v_2 = v_1}$

- Eliminates useless recalculation.
- Paves the way for later optimizations.

Copy Propagation

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_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp6) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp3 ;
_tmp4 = _tmp3 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp0 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = _tmp0 ;
a = _tmp0 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = c ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam c ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Copy Propagation

- If we have a variable assignment

  $v_1 = v_2$

  then as long as $\mathbf{v_1}$ and $\mathbf{v}_2$ are not reassigned, we can rewrite expressions of the form

  $a = \ldots v_1 \ldots$

  as

  $a = \ldots v_2 \ldots$

  provided that such a rewrite is legal.
- This will help immensely later on, as you'll see.

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp3 = 4 ;
a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


a = 4 ;
_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;
c = _tmp4 ;
_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;


_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;


_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;


_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;



_tmp4 = _tmp0 + b ;



_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

- An assignment to a variable **v** is called **dead** if the value of that assignment is never read anywhere.
- **Dead code elimination** removes dead assignments from IR.
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned.

For Comparison

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
_tmp4 = _tmp0 + b ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Applying Local Optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization.
  - Common subexpression elimination eliminates unnecessary statements.
  - Copy propagation helps identify dead code.
  - Dead code elimination removes statements that are no longer needed.
- To get maximum effect, we may have to apply these optimizations numerous times.

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Other Types of Local Optimization

- **Arithmetic Simplification**
  - Replace “hard” operations with easier ones.
  - e.g. rewrite **`x = 4 * a`**; as **`x = a << 2`**;
- **Constant Folding**
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite **x = 4 * 5;** as **x = 20;**.

Implementing Local Optimization

Optimizations and Analyses

- Most optimizations are only possible given some analysis of the program's behavior.
- In order to implement an optimization, we will talk about the corresponding program analyses.

Available Expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the **available expressions** in a program.
- An expression is called **available** if some variable in the program holds the value of that expression.
- In common subexpression elimination, we replace an available expression by the variable holding its value.
- In copy propagation, we replace the use of a variable by the available expression it holds.

Finding Available Expressions

- Initially, no expressions are available.
- Whenever we execute a statement
  a = b + c:
  - Any expression holding **a** is invalidated.
  - The expression **a = b + c** becomes available.
- **Idea**: Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable.

Available Expressions

```
a = b;

c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

{ }

```
a = b;

c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
```

Available Expressions

{ }

```
a = b;
```

{ a = b }

```
c = b;
```

{ a = b, c = b }

```
d = a + b;
```

{ a = b, c = b, d = a + b }

```
e = a + b;
```

{ a = b, c = b, d = a + b, e = a + b }

```
d = b;
```

{ a = b, c = b, d = b, e = a + b }

```
f = a + b;
```

{ a = b, c = b, d = b, e = a + b, f = a + b }

Common Subexpression Elimination

{ }

```
a = b;
```

{ a = b }

```
c = b;
```

{ a = b, c = b }

```
d = a + b;
```

{ a = b, c = b, d = a + b }

```
e = a + b;
```

{ a = b, c = b, d = a + b, e = a + b }

```
d = b;
```

{ a = b, c = b, d = b, e = a + b }

```
f = a + b;
```

{ a = b, c = b, d = b, e = a + b, f = a + b }

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

{ }

a = b;

{ a = b }

c = **a**;

{ a = b, c = b }

d = a + b;

{ a = b, c = b, d = a + b }

e = **d**;

{ a = b, c = b, d = a + b, e = a + b }

d = **a**;

{ a = b, c = b, d = b, e = a + b }

f = **e**;

{ a = b, c = b, d = b, e = a + b, f = a + b }

Common Subexpression Elimination

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

Live Variables

- The analysis corresponding to dead code elimination is called **liveness analysis**.
- A variable is **live** at a point in a program if later in the program its value will be read before it is written to again.
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a = b + c**:
  - Just before the statement, **a** is not alive, since its value is about to be overwritten.
  - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
  - *(what if we have* **a = a + b***?)*

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
c = a;
d = a + b;
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;
```
{ a, b, d }
```
e = d;
```
{ a, b, e }
```
d = a;
```
{ b, d, e }
```
f = e;
```
{ b, d }

Liveness Analysis

```
a = b;

c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }

{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;


d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;



d = a + b;

e = d;

d = a;
{ b, d }
```

Liveness Analysis II

```
a = b;
```

```
d = a + b;

e = d;
```

{ a, b }

```
d = a;
```

{ b, d }

Liveness Analysis II

```
a = b;

d = a + b;
```
{ a, b, d }
```
e = d;
```
{ a, b }
```
d = a;
```
{ b, d }

Liveness Analysis II

```
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }

{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;
```

```
d = a + b;
```

```
d = a;
```

{b, d}

Liveness Analysis III

```
a = b;
```

```
d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

Liveness Analysis III

```
a = b;
```

{a, b}

```
d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

Liveness Analysis III

{b}
```
a = b;
```
{a, b}
```
d = a + b;
```
{a, b}
```
d = a;
```
{b, d}

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}



{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
a = b;
```

```
d = a;
```

A Combined Algorithm

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
{b, d}
```

A Combined Algorithm

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
{a, b}
d = a;



{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;


{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;

c = a;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;



{a, b}
d = a;


{b, d}
```

A Combined Algorithm

```
a = b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
{b}
a = b;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;
```

```
d = a;
```